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IFS EDUCATION MODEL DOCUMENTATION

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1. Introduction

Education is paramount to human and societal development. Yet many countries fail to provide the opportunity of education for all citizen. What are the socio-economic consequences of following this low-education path? How much resources do the societies require to sustain and/or expand their educational participation and progression rates? What is the level of educational attainment of the people in a society? How does that attainment impact the economic and demographic outcomes in the society? What kind of resources are required to move attainment? What is the payoff horizon of such attainment? Seeking interactive answers to such education policy questions require a model that can study the dynamics of the education sector in the broader context of economic activities, societal transitions and governance decisions.

International Futures (IFs) Model is an integrated global computer simulation attempting to understand multiple cross-cutting issues areas including education and to explore possible actions that can help countries change course. This document describes and explains the International Futures education model.

IFs education model forecasts enrollment, financing and attainment of education in 186 countries. It covers formal education spanning elementary, lower secondary, upper secondary and tertiary. It forecasts intake, survival, graduation and transition rates for each of these levels separately for boys and girls. At the elementary level the model distinguishes between the of-age and over-age pupils by computing a net rate and a gross rate for entrance and enrollment. Secondary education is disaggregated in the model into lower and upper secondary each of which are further divided into general and vocational programs. In addition to college graduates, higher education model also computes science and engineering graduates. On the financial side, the model compute total and per student spending at each level. Educational attainment variables computed in the model are the level of education completed and the average years of education acquired by the people grouped into five-year age-sex cohorts. The national education systems simulated in the model follow UNESCO's ISCED classification system of levels of education and are thus roughly comparable even though the entrance ages or cycle lengths can slightly differ among countries. The model runs recursively in annual time steps for a horizon that can be extended to the end of the twenty first century.

The modeling methodology centers on a stock and flow accounting mechanism that tracks the flow of children into, across and out of the stocks of pupils. The rates of flow are determined by the secular trend of increasing education with increased level of development, the fiscal constraints and the growth and saturation of rates as the economic and financial constraints are lifted. As the boys and girls leave school they carry along the acquired education and the total stock of attained education is adjusted accordingly.

The education model is developed as a sub-model of the International Futures (IFs) World Model. Among the other IFs sub-models are - population, economy, government finance, infrastructure, energy, health, governance and environment. Each of these models simulate the complex interactions in one of the major human or natural systems and together they paint a comprehensive picture of the key dynamics within and across these systems. The IFs models that are most closely linked with the IFs education model are the demographic model, the economic model and the model that represents government finance. The causal relationships simulated by the models are often bi-directional, implemented through a combination of analytical functions, table functions and various feedback algorithm. The example of such a bi-directional linkage is the relationship between education and demography. On one hand, population of the corresponding age groups, computed in the demographic model, are multiplied with student flow rates to determine student headcount, on the other, education of women is one of the various drivers of fertility rate. In a similar feedback relationship, additional investment in education, assuming there is enough demand and no waste, would result in higher completion rates and more educational attainment. The extra attainment would ultimately boost productivity, as the better educated youth join the workforce, and make it possible to invest more in education.

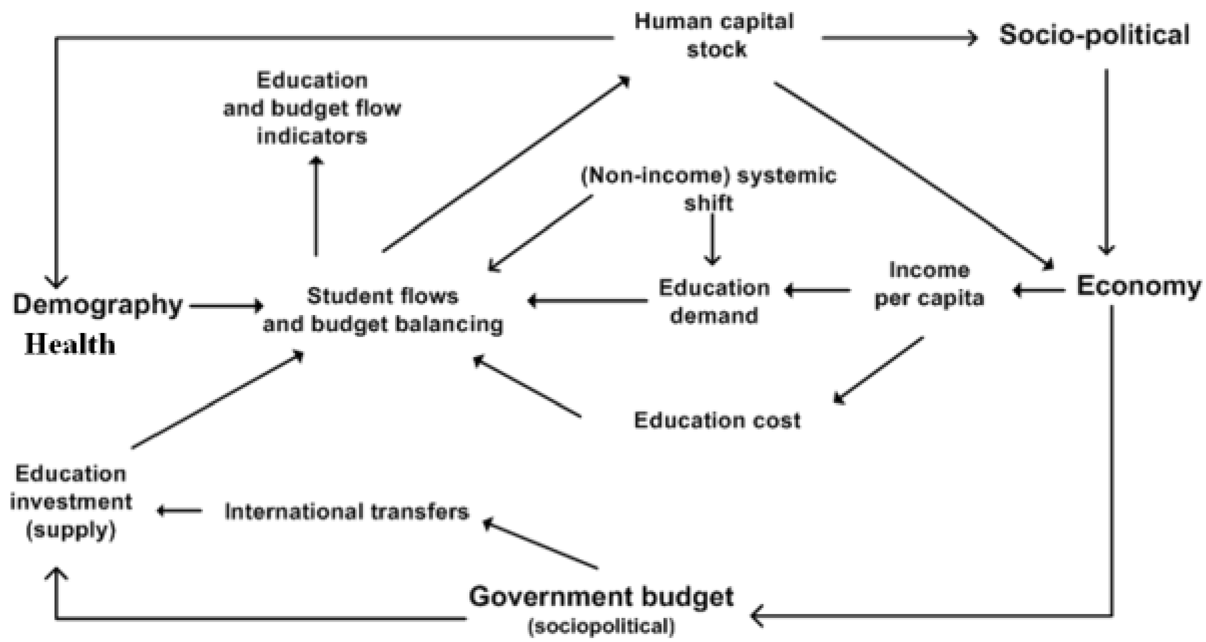
IFs education model is not a novel attempt in building a global multi-country education forecasting. Researchers have developed education models for projecting enrollment (Wils and O'Connor 2003), costs (Delamonica, Mehrotra and Vandemoortele 2001; Bruns, Mingat and Rakotomalala 2003), attainment (KC et al 2010) and impacts of education (McMahon 1999, KC et al 2010). Most of these models project enrollment through trend extrapolation or a causal relationship working directly on enrollment. In reality, enrollment is a stock that can change only through inflows and/or outflows. IFs education model represents this stock and flow structure as faithfully as possible by imposing the causal relationship only on the flow rates like entrance or survival and computing enrollment through the accounting process. The flow rates themselves are connected to the fundamentals in an endogenous model. The model tracks and connects the educational efforts and attainment throughout the lifecycle of a person. The single-year age cohorts computed in the IFs population model makes the simulation possible. Public financing of education is integrated with the government budget process simulated in the IFs Government finance model. The educational investment, in the model, brings in economic and social returns at the national and the global level, explicitly or implicitly, as in the case of global impacts. At the societal level, it attempts to simulate the interactions of education with the broader society in an endogenous framework. The long run-horizon of the International Futures modeling platform makes it possible for a model user to estimate the full returns of investment in education realized over multiple generations. The model includes parameters, in all three areas - enrollment, attainment and financing – making it possible to develop alternative scenarios to explore uncertainties and to analyze policy interventions. The model thus serves as a generalized thinking and analysis tool for educational futures within a broader human development context.

1.1 Conceptual Framework

The next figure lays out the conceptual framework of the IFs education model. The figure shows:

a. Major algorithmic pieces inside the education module, e.g. student flows, budget balancing
b. Other IFs models that drive or are driven by education variables, and
c. the causal connections with the direction of causation.

In the center of the figure we have the student flow and budget balancing piece, the core pieces of our education model. On the two sides, we have Economy and Demography, the non-educational models of highest relevance to education. Income per capita, widely used as an indicator for the level of development of a society, computed in the Economy model of IFs determine rates of entrance, persistence and transition. IFs cohort-specific demographic model provides the school age population to the education model. The enrollment counts are obtained by multiplying the population with the flow rates. These enrollments are multiplied with per student cost, which is also driven by the level of income. The demand for funds is sent to the government finance model. Domestic revenue and international transfers, computed in the economic model, together form the total public funds available. The distribution of budget among education and other public spending sectors takes place in the government finance model. The stock of human capital (i.e., educational attainment of adults) gets updated in the model as the children reach adulthood taking their educational accomplishment with them. Education of people impact fertility, mortality and nutrition in the demographic and health models. In the IFs model of economy, productivity is driven by human capital. Several variables in the IFs governance model, for example state stability and democracy, also use education as a driver.



1.2 Dominant Relations: Education

The dominant relationships in the IFs Education model are those that determine various educational flow rates, e.g., intake rate for primary (EDPRIINT) or tertiary (EDTERINT), or survival rates in primary (EDPRISUR) or lower secondary (EDSECLOWRSUR). These rates are functions of per capita income. Non-income drivers of education are represented by upward shifts in these functions. These rates follow an S-shaped path in most cases. The flows interact with a stocks and flows structure to derive major stocks like enrollment, for the young, and attainment, for the adult.

On the financing side, the major dynamic is in the cost of education, e.g., cost per student in primary, EDEXPERPRI, the bulk of which is teachers' salary and which thus goes up with rising income.

Public spending allocation in education, GDS(Educ) is a function of national income per capita that proxies level of economic development. Demand for educational spending - determined by initial projections of enrollment and of per student cost - and total availability of public funds affect the base allocation derived from function.

For diagrams see: Education: Student Flow; Education Budget Flow

For equations see: Student flow equation Budget Equations

1.3 Key dynamics are directly linked to the dominant relations

- Intake, survival and transition rates are functions of per capita income (GDPPCP). These functions shift upward over time representing the non-income drivers of education.
- Each year flow rates are used to update major stocks like enrollment, for the young, and attainment, for the adult.
- Per student expenditure at all levels of education is a function of per capita income.
- Deficit or surplus in public spending on education, GDS(Education), affects intake, transition and survival rates at all levels of education.

1.4 Structure and Agent System: Education

Formal Education

System/Sub System	Formal Education (elementary, lower secondary, upper secondary and tertiary)
Organizing Structure	Grade-flow model, i.e., entrance and progression of children from one grade to the next and transition from one level to the next.
Stocks	Students, children
Flows	Entrants, graduates, drop-outs
Key Aggregate Relationships	Access, participation and progress rates move with the level of development of the society. Rates of change for the boys and girls different with the boys gaining more access and progression at the earlier levels. Rates also vary by level of education. Higher levels, understandably, move much slower than the more basic levels. Educational flow curves shift upward in the long run as countries move towards a more knowledge based society. The model implements a systemic shift for some of the curves.

Key Agent-Class Behavior Relationships	Individuals and families decision to pursue education

Education Finance

System/Sub System	Government spending by destination
Organizing Structure	Normalization of budget share given the overall budget constraint and the emphasis on education
Stocks	Per student cost for different levels of schooling
Flows	Public spending in education
Key Aggregate Relationships	Public spending available for education rises with the growth in revenue collection that moves with the level of development. Cost of education rises with the income level in the country. Demand for public funds in education grows with the growth in costs and/or the growth in school-age population. Education budget competes with other sectors of government expenditure. Enrollment and completion rates are affected by funding decisions. Educational budget push can affect other development priorities.
Key Agent-Class Behavior Relationships	Government revenue, expenditure and transfer payment

Educational Attainment

System/Sub System	Educational attainment of adults
Organizing Structure	Distribution of population by age, sex and educational attainment
Stocks	Population, level and years of education obtained
Flows	Completion, drop-out, deaths, births, aging
Key Aggregate Relationships	With the rise of enrollment and completion attainment level change, first for the young adults and, somewhat slowly, for the overall population.
Key Agent-Class Behavior Relationships	Higher level of attainments boosts economic productivity. Level of education of women affect fertility rates.

2 Concepts and Coverage

2.1 National Education System

UNESCO has developed a standard classification system for national education systems called International Standard Classification of Education (ISCED). ISCED 2011[1], which evolved from the earlier ISCED 1997, uses a numbering system to identify the sequential levels of educational systems—namely, pre-primary, primary, lower secondary, upper secondary, post-secondary non tertiary and tertiary—which are characterized by curricula of increasing difficulty and specialization as the students move up the levels. IFs education model covers primary (ISCED level 1), lower secondary (ISCED level 2), upper secondary (ISCED level 3), and tertiary education (ISCED levels 5A, 5B and 6).

The model covers 186 countries that can be grouped into any number of flexible country groupings, e.g., UNESCO regions, like any other sub-module of IFs. Country specific entrance age and school-cycle length data are collected and used in IFs to represent national education systems as closely as possible. For all of these levels, IFs forecast variables representing student flow rates, e.g., intake, persistence, completion and graduation, and stocks, e.g., enrolment, with the girls and the boys handled separately within each country.

[1] Please check http://uis.unesco.org/en/isced-mappings for more details on ISCED 2011 classification system

For lower and upper secondary, the IFs model covers both general and vocational curriculum and forecasts the vocational share of total enrolment, EDSECLOWRVOC (for lower secondary) and EDSECUPPRVOC (for upper secondary). Like all other participation variables, these two are also disaggregated by gender. IFs model of tertiary education computes science and engineering graduates, separately, in addition to all college graduates, higher education model

2.2 Student Flow Rates

Educational databases express flows of students through education systems as various rates of flow, for example, intake rate or enrollment rate. These rates are not rates of change over time. They are rather student shares of a relevant population group expressed as a percentage. Depending on the particular rate of flow, the population used to compute the percentage share could be:

- a single-year age-cohort, for example, number of new entrants in the first grade of primary expressed as a percentage of population at the official entrance age of primary gives the intake rate for primary (this is actually a net intake rate as opposed to a gross rate, a distinction that we will explain soon). Intake and graduation rates at all levels use single-year age cohort.
- a multi-year age cohort, for example, elementary enrollment rate is computed from the number of primary pupils and the total population of the official age group corresponding to the first to the final grade of primary. Enrollment rates need multi-year cohorts.
- a cohort of students, for example, survival rates in primary are computed as the percentage of first graders who persist till the final grade of elementary. Another example in this group is transition rate, which is expressed as a percentage of graduates in one level who enter the first grade of the next level in a subsequent year.

Another important distinction among the flow rates is a gross rate versus a net rate, applicable to some of the flows. The need for this distinction comes from the phenomenon of over-age (and in some richer society cases, under-age) entrance and enrollment which could be substantial in low-education countries in a catch up phase. **Gross rates** include all pupils or entrants, regardless of age, whereas **net rates** include only those who are of the official age (or age group). All of the flow rates forecast in the IFs education model are gross rates except three: entrance (or intake) and enrollment rates in primary and enrollment rate in total secondary. This distinction does not apply to survival or transition rates because of the way those variables are defined.

2.3 Attainment

The output of the national education system, i.e., school completion and partial completion of the young people, is added to the educational attainment of the adults in the population. IFs forecasts four categories of attainment - portion with no education, completed primary education, completed secondary education and completed tertiary education - separately for men and women above fifteen years of age by five year cohorts as well as an aggregate over all adult cohorts. The model software contains so-called "Education Pyramid," a display of educational attainments mapped over five-year age cohorts by sex as is usually done for population pyramids.

Another aggregate measure of educational attainment that we forecast is the average years of education of the adults. We have several measures, EDYEARSAG15, average years of education for all adults aged 15 and above, EDYRSAG25, average years of education for those 25 and older, EDYRSAG15TO24, average years of education for the youngest of the adults aged between fifteen years to twenty-four.

2.4 Finance

IFs education model also covers financing of education. The model forecasts per student public expenditure as a share of per capita income. The model also forecasts total public spending in education and the share of that spending that goes to each level of education.

2.5 What the Model Does Not Cover

ISCED level 0, pre-primary, and level 4, post-secondary pre tertiary, are not common across all countries and are thus excluded from the IFs education model which has a global coverage.

On the financing side, the model does not include private spending in education, a significant share of spending especially for tertiary education in many countries and even for secondary education in some countries. Scarcity of good data and lack of any pattern in the available data precludes modelling private spending in education.

Quality of national education system can also vary across countries and over time. The IFs education model does not forecast any explicit indicator of education quality. However, the survival and graduation rates that the model forecasts for all levels of education are implicit indicators of system quality. At this point IFs does not forecast any indicator of cognitive quality of learners. However, the IFs database does have data on cognitive quality.

2.6 Variable Naming Convention

All education model variable names start with a two-letter prefix of 'ED' followed, in most cases, by the three letter level indicator - PRI for primary, SEC for secondary, TER for tertiary. Secondary is further subdivided into SECLOWR for lower secondary and SECUPPR for upper secondary. Parameters in the model, which are named using lowercase letters like those in other IFs modules, also follow a similar naming convention.

3 Education Data

An historical database plays an important role in the operationalization of a conceptual model. The ongoing convergence of formal educational standards around the world made it easier for international agencies and researchers to develop international educational standards and collect comparable data with global coverage. This section describes how we have gathered and used such data.

3.1 Education Data Sources

Data used in the IFs education model comes from international development agencies with global or regional coverage, policy think-tanks and academic researchers. Some of these data are collected through census and survey of educational institutes conducted by national governments and reported to international agencies. Some data are collected through household surveys. In some cases, data collected through survey and census are processed by experts to create internationally comparable data sets.

UNESCO, the UN agency charged with collecting and maintaining education-related data from across the world, is the primary source for the education data we use in the model. UNESCO local offices collect the data by working with country governments. UNESCO Institute for Statistics (UIS, http://uis.unesco.org/) publish global time series in their online data repository whence we get the data.

World Bank's World Development Indicator (WDI) database (http://data.worldbank.org/data-catalog/world-development-indicators) incorporates major educational series from UIS. The World Bank also maintains its own online educational database titled EdStats (http://datatopics.worldbank.org/education/). EdStats has recently started adding data on educational equality.

We would also like to mention some other international education database from which we do not yet use any data in our model. UNICEF collects education data from households through their Multiple Indicator Cluster Survey(MICS). Household level data is also collected by USAID as a part of its Demographic and Household Surveys (DHS). Organization for Economic Cooperation and Development (OECD), an intergovernmental organization of rich and developed economies host an online education database at http://www.oecd.org/education/database.htm. Their data covers thirty-five member countries and some non-members (Argentina, Brazil, China, India Colombia, Costa Rica, Indonesia, Lithuania, Russia, Saudi Arabia and South-Africa are some of the non-members covered in the OECD database). OECD also publish an annual compilation of indicators titled Education at a Glance (http://www.oecd.org/edu/education-at-a-glance-19991487.htm). OECD's data include education quality data in the form of internationally administered assessment tests. Several other regional agencies, for example, Asian Development Bank or EU's Eurostat also publish educational data as a part of their larger statistical efforts.

Research organizations and academic researchers sometime compute education data not available through survey and census, but can be computed from those. For example, the educational attainment dataset compiled by Robert Barro and Jong Wha Lee (2013) is widely used. International Institute for Applied Systems Analysis (IIASA) did also compile attainment data using household survey data obtained from MICS and DHS surveys. Global Monitoring Report team of UNESCO computes educational inequalities within and across countries and publish them in a database titled World Inequality Database on Education (http://www.education-inequalities.org/).

3.2 Processing Education Data in IFs

Enrollment, attainment and financing data that we collect from various sources are utilized in two ways. First, data help us operationalize the dominant model relations by estimating the direction, magnitude and strength of the relationship. Second, data is used for model initialization as described in the next section.

3.2.1 Model Initialization

IFs education model, like all other IFs models, is a recursive dynamic model running in annual time steps. Model initialization is handled in a preliminary process in which model variables are assigned values for the starting year of the model's run-horizon. The model pre-processor serves two purposes. First, initialization with the most recent data ensures continuity between the real world developments and the forecast. Second, inconsistency between historical data and model equations are removed in the pre-processor through various reconciliation procedures.

3.2.2 Data Reconciliation

Inconsistencies among the base year primary flow rates (intake, survival, and enrollment) can arise either from reported data values that, in combination, do not make sense, or from the use of "stand-alone" cross-sectional estimations used in the IFs pre-Processor to fill missing data. Such incongruities might arise among flow rates within a single level of education (e.g., primary intake, survival, and enrollment rates that are incompatible) or between flow rates across two levels of education (e.g., primary completion rate and lower secondary intake rate).

The IFs education model uses algorithms to reconcile incongruent flow values. They work by (1) analyzing incongruities; (2) applying protocols that identify and retain the data or estimations that are probably of higher quality; and (3) substituting recomputed values for the data or estimations that are probably of lesser quality. For example, at the primary level, data on enrollment rates are more extensive and more straight-forward than either intake or survival data; in turn, intake rates have fewer missing values and are arguably more reliable measures than survival rates. The IFs pre-processor reconciles student flow data for Primary by using an algorithm that assumes enrollment numbers to be more reliable than the entrance data and entrance data to be more reliable than survival data.

4 Education Flow Charts

4.1 Education Overview

For each country, the IFs education model represents a multilevel formal education system that starts at primary and ends at tertiary. Student flows, i.e., entry into and progression through the system are determined by forecasts on intake and persistence (or survival) rates superimposed on the population of the corresponding age cohorts obtained from IFs population forecasts. Students at all levels are disaggregated by gender. Secondary education is further divided into lower and upper secondary, and then further into general and vocational according to the curricula that are followed.

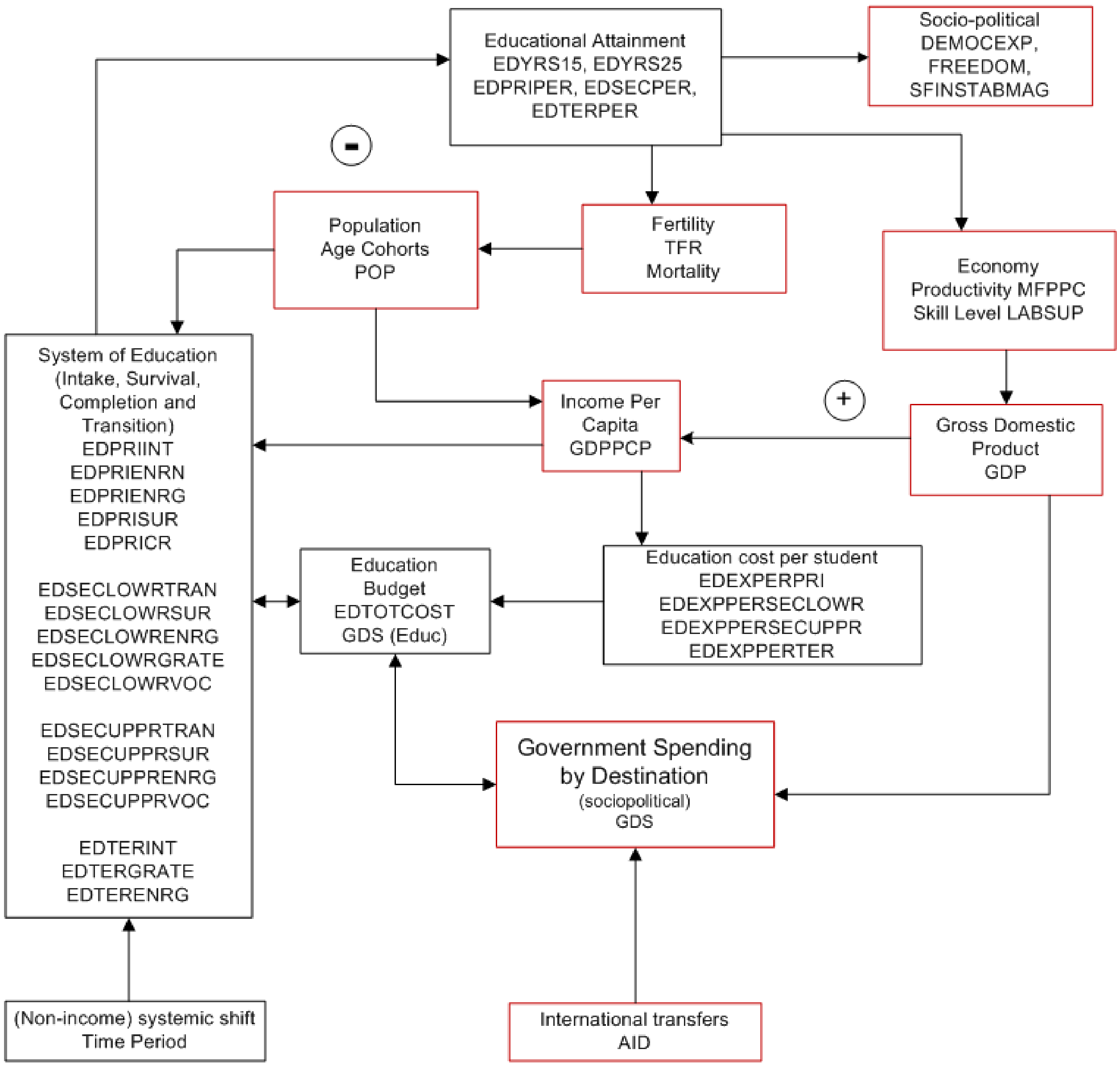
The model represents the dynamics in education financing through per student costs for each level of education and a total public spending in education. Policy levers are available for changing both spending and cost.

School completion (or dropout) in the education model is carried forward as the educational attainment of the overall population. As a result, the education model forecasts population structures by age, sex, and attained education, i.e., years and levels of completed education.

The major agents represented in the education system of the model are households,—represented by the parents who decide which of their boys and girls will go to school—and governments that direct resources into and across the educational system. The major flows within the model are student and budgetary, while the major stock is that of educational attainment embedded in a population. Other than the budgetary variables, all the flows and stocks are gender disaggregated.

The education model has forward and backward linkages with other parts of the IFs model. During each year of simulation, the IFs cohort-specific demographic model provides the school age population to the education model. In turn, the education model feeds its calculations of education attainment to the population model's determination of women's fertility. Similarly, the broader economic and socio-political systems provide funding for education, and levels of educational attainment affect economic productivity and growth, and therefore also education spending.

The figure below shows the major variables and components that directly determine education demand, supply, and flows in the IFs system. The diagram attempts to emphasize on the inter-connectedness of the education model components and their relationship to the broader human development system.

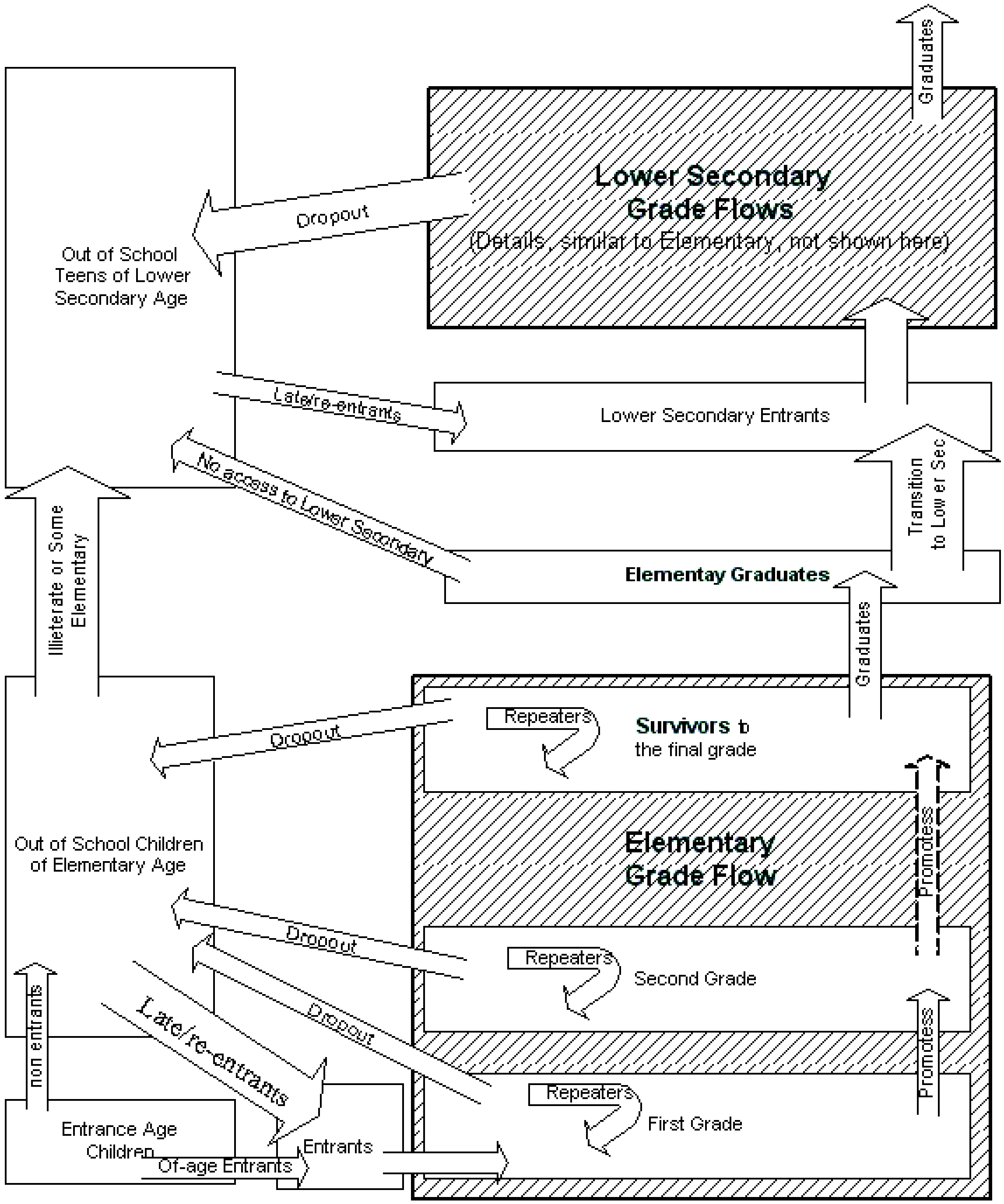


4.2 Education Student Flow

IFs education model simulates grade-by-grade student flow for each level of education that the model covers. Grade-by-grade student flow model combine the effects of grade-specific dropout, repetition and reentry into an average cohort-specific *grade-to-grade flow rate*, calculated from the survival rate for the cohort. Each year the number of new entrants is determined by the forecasts of the intake rate and the entrance age population. In successive years, these entrants are moved to the next higher grades, one grade each year, using the *grade-to-grade flow rate*. The simulated grade-wise enrollments are then used to determine the total enrollment at the particular level of education. Student flow at a particular level of education, e.g., primary, is culminated with rates of completion and transition by some to the next level, e.g., lower secondary.

The figure below shows details of the student flow for primary (or, elementary) level. This is illustrative of the student flow at other levels of education. We model both net and gross enrollment rates for primary. The model tracks the pool of potential students who are above the entrance age (as a result of never enrolling or of having dropped out), and brings back some of those students, marked as late/reentrant in the figure, (dependent on initial conditions with respect to gross versus net intake) for the dynamic calculation of total gross enrollments.

A generally similar grade-flow methodology models lower and upper secondary level student flows. We use country-specific entrance ages and durations at each level. As the historical data available does not allow estimating a rate of transition from upper secondary to tertiary, the tertiary education model calculates a tertiary intake rate from tertiary enrollment and graduation rate data using an algorithm which derives a tertiary intake with a lower bound slightly below the upper secondary graduation rate in the previous year.

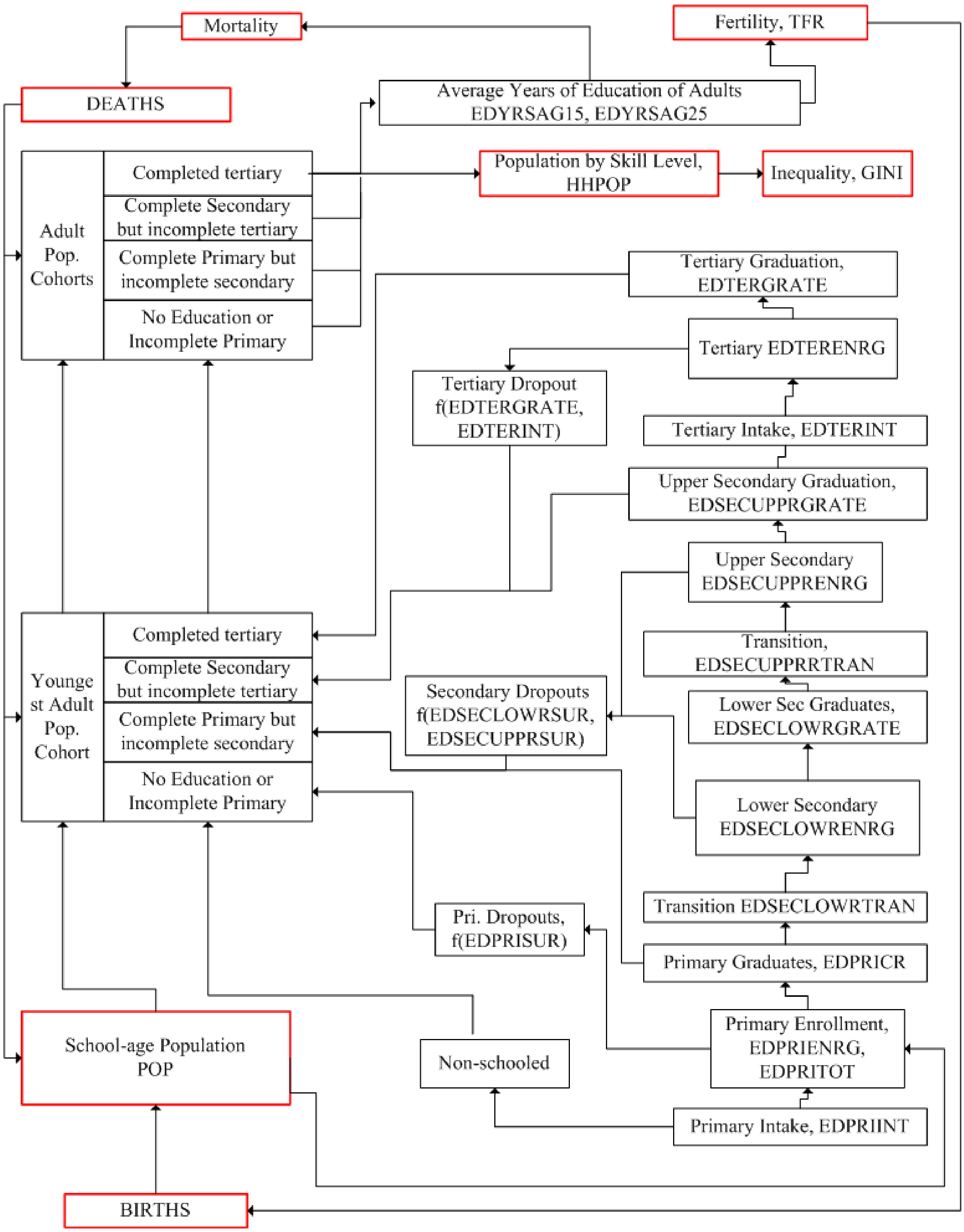


4.3 Education Attainment

The algorithm for the tracking of education attainment is very straight-forward. The model maintains the structure of the population not only by age and sex categories, but also by years and levels of completed education. In each year of the model's run, the youngest adults pick up the appropriate total years of education and specific levels of completed education. The model advances each cohort in 1-year time steps after subtracting deaths. In addition to cohort attainment, the model also calculates overall attainment of adults (15+ and 25+) as average years of education (EDYRSAG15, EDYRSAG25) and as share of people 15+ with a certain level of education completed (EDPRIPER, EDSECPER, EDTERPER).

One limitation of our model is that it does not represent differential mortality rates associated with different levels of education attainment (generally lower for the more educated). [2] This leads, other things equal, to a modest underestimate of adult education attainment, growing with the length of the forecast horizon. The averaging method that IFs uses to advance adults through the age/sex/education categories also slightly misrepresents the level of education attainment in each 5-year category.

[2] The multi-state demographic method developed and utilized by IIASA does include education-specific mortality rates.


Mortality
Fertility, TFR
DEATHS
Average Years of Education of Adults
EDYRSAG15, EDYRSAG25
Adult
Pop.
Cohorts
Completed tertiary
Complete Secondary
but incomplete tertiary
Complete Primary but
incomplete secondary
No Education or
Incomplete Primary
Population by Skill Level,
HHPOP
Inequality, GINI
Tertiary Graduation,
EDTERGRATE
Tertiary EDTERENRG
Tertiary Dropout
f(EDTERGRATE,
EDTERINT)
Tertiary Intake, EDTERINT
Upper Secondary Graduation,
EDSECUPPRGRATE
Upper Secondary
EDSECUPPRENRG
Younge
st Adult
Pop.
Cohort
Completed tertiary
Complete Secondary
but incomplete tertiary
Complete Primary but
incomplete secondary
No Education or
Incomplete Primary
Transition,
EDSECUPPRRTRAN
Secondary Dropouts
f(EDSECLOWRSUR,
EDSECUPPRSUR)
Lower Sec Graduates,
EDSECLOWRGRATE
Lower Secondary
EDSECLOWRENRG
Transition EDSECLOWRTRAN
Pri. Dropouts,
f(EDPRISUR)
Primary Graduates, EDPRICR
School-age Population
POP
Non-schooled
Primary Enrollment,
EDPRIENRG,
EDPRITOT
Primary Intake, EDPRIINT
BIRTHS

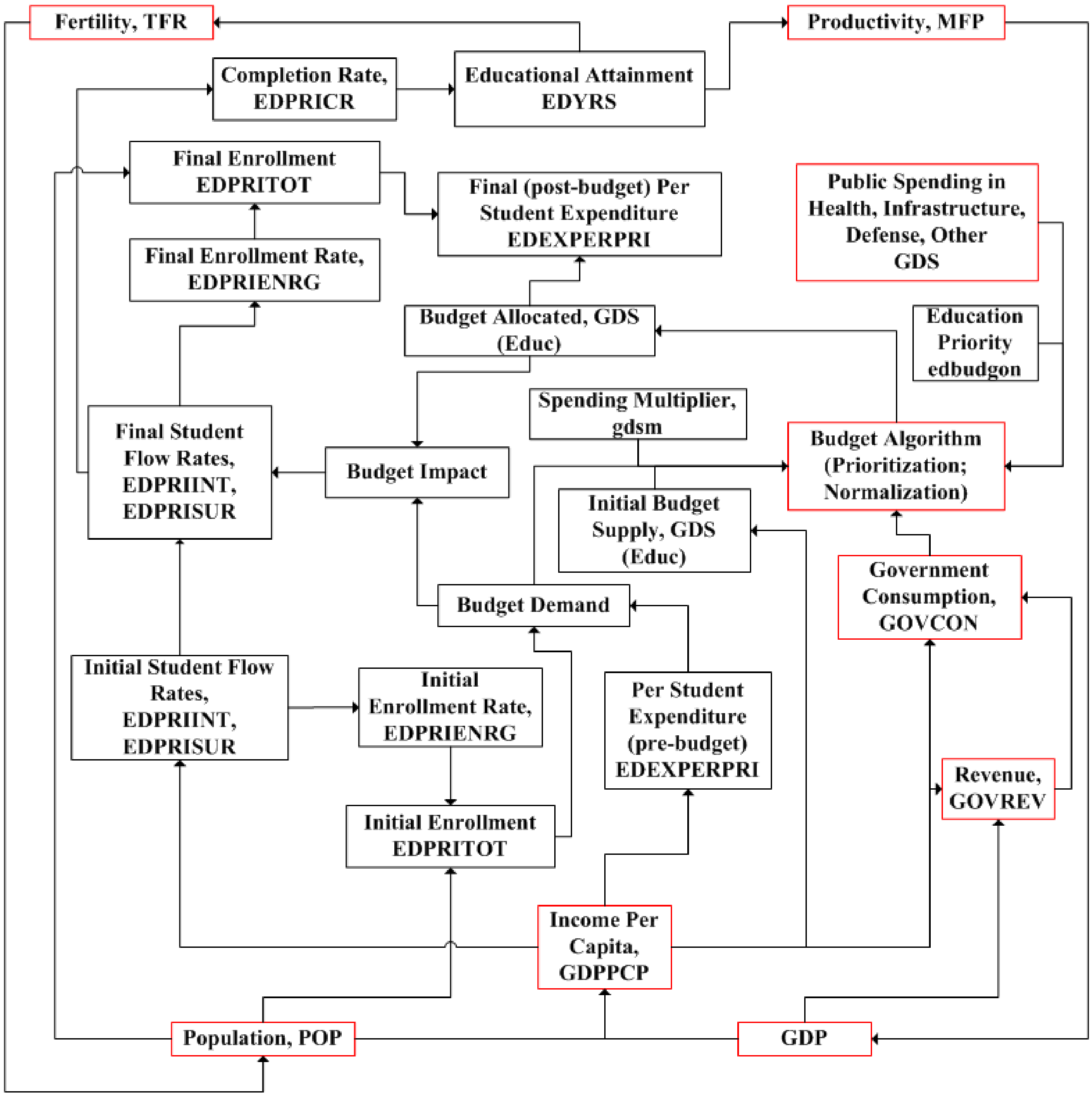
4.4 Education Financial Flows

In addition to student flows, and interacting closely with them, the IFs education model also tracks financing of education. Because of the scarcity of private funding data, IFs specifically represents public funding only, and our formulations of public funding implicitly assume that the public/private funding mix will not change over time.

The accounting of educational finance is composed of two major components, per student cost and the total number of projected students, the latter of the two is discussed in the student flows section. Spending per student at all levels of education is driven by average income. Given forecasts of spending per student by level of education and given initial enrollments forecasts by level, an estimate of the total education funding demanded is obtained by summing across education levels the products of spending per student and student numbers.

The funding needs are sent to the IFs government finance model where educational spending is initially determined from the patterns in such spending regressed against the level of economic development of the countries. A priority parameter (**edbudgon**) is then used to prioritize spending needs over spending patterns. This parameter can be changed by model user within a range of values going from zero to one with the zero value awarding maximum priority to fund demands. Finally, total government consumption spending (GOVCON) is distributed among education and other social spending sectors, namely infrastructure, health, public R&D, defense and an "other" category, using a normalization algorithm.

Government spending is then taken back to the education module and compared against fund needs. Budget impact, calculated as a ratio of the demanded and allocated funds, makes an impact on the initial projection of student flow rates (intake, survival, and transition). The positive (upward) side of the budget impact is non-linear with the maximum boost to growth occurring when a flow rate is at or near its mid-point or within the range of the inflection points of an assumed S-shaped path, to be precise. Impact of deficit is more or less linear except at impact ratios close to 1, whence the downward impact is dampened. Final student flow rates are used to calculate final enrollment numbers using population forecasts for relevant age cohorts. Finally, cost per students are adjusted to reflect final enrollments and fund availability.



For the sake of simplicity, only elementary education variables are used

5 Education Equations

5.1 Education Equations Overview

The IFs education model represent two types of educational stocks, stocks of pupils and stocks of adults with a certain level of educational attainment. These stocks are initialized with historical data. The simulation model then recalculates the stock each year from its level the previous year and the net annual change resulting from inflows and outflows.

The core dynamics of the model is in these flow rates. These flow rates are expressed as a percentage of age-appropriate population and thus have a theoretical range of zero to one hundred percent. Growing systems with a saturation point usually follow a sigmoid (S-shaped) trajectory with low growth rates at the two ends as the system begins to expand and as it approaches saturation. Maximum growth in such a system occurs at an inflection point, usually at the middle of the range or slightly above it, at which growth rate reverses direction. Some researchers (Clemens 2004; Wils and O'Connor 2003) have identified sigmoid trends in educational expansion by analyzing enrollment rates at elementary and secondary level. The IFs education model is not exactly a trend extrapolation; it is rather a forecast based on fundamental drivers, for example, income level. Educational rates in our model are driven by income level, a systemic shift algorithm and a budget impact resulting from the availability of public fund. However, there are growth rate parameters for most of the flows that allow model user to simulate desired growth that follows a sigmoid-trajectory. Another area that makes use of a sigmoid growth rate algorithm is the boost in flow rates as a result of budget surplus.

Intake (or transition), survival, enrollment and completion are some of the rates that IFs model forecast. Rate forecasts cover elementary, lower secondary, upper secondary and tertiary levels of education with separate equations for boys and girls for each of the rate variables. All of these rates are required to calculate pupil stocks while completion rate and dropout rate (reciprocal of survival rate) are used to determine educational attainment of adults.

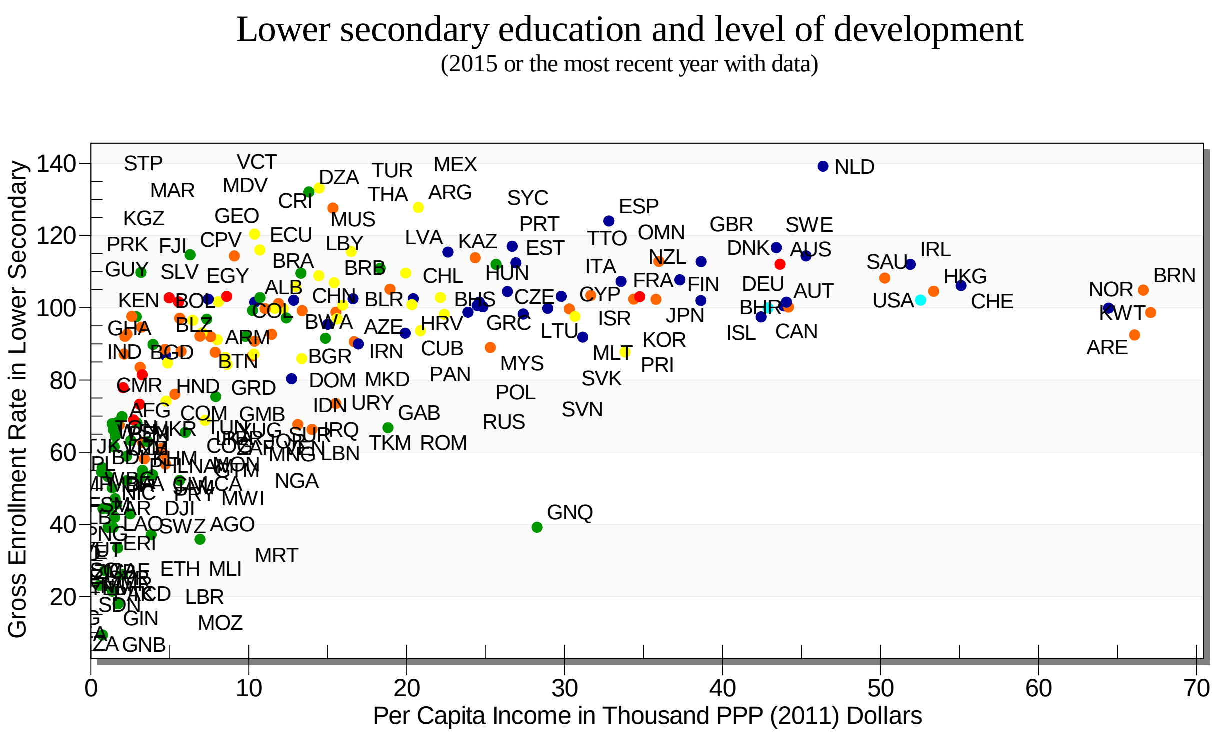
On the financial side of education, IFs forecast cost per student for each level. These per student costs are multiplied with enrollments to calculate fund demand. Budget allocation calculated in IFs socio-political module is sent back to education model to calculate final enrollments and cost per student as a result of fund shortage or surplus.

The population module provides cohort population to the education model. The economic model provides per capita income and the socio-political model provides budget allocation. Educational attainment of adults calculated by the education module affects fertility and mortality in the population and health modules, affects productivity in the economic module and affects other socio-political outcomes like governance and democracy levels.

5.2 Education Equations: Student Flow: Regression Models for Core Flow Rates

Enrollments at various levels of education - EDPRIENRN, EPRIENRG, EDSECLOWENRG, EDSECUPPRENRG, EDTERENRG - are initialized with historical data for the beginning year of the model. Net change in enrollment at each time step is determined by inflows (intake or transition) and outflows (dropout or completion). Entrance to the school system (EDPRIINT, EDTERINT), transition from the lower level (EDSECLOWRTRAN, EDSECUPPRTRAN) - and outflows - completion (EDPRICR), dropout or it's reciprocal, survival (EDPRISUR) - are some of these rates that are forecast by the model.

The educational flow rates are best explained by per capita income that serves as a proxy for the families' opportunity cost of sending children to school. For each of these rates, separate regression equations for boys and girls are estimated from historical data for the most recent year. These regression equations, which are updated with most recent data as the model is rebased with new data every five years, are usually logarithmic in form. The following figure shows such a regression plot for net intake rate in elementary against per capita income in PPP dollars.



In each of the forecast years, values of the educational flow rates are first determined from these regression equations. Independent variables used in the regression equations are endogenous to the IFS model. For example, per capita income, GDPPCP, forecast by the IFs economic model drives many of the educational flow rates. The following equation[3] shows the calculation of one such student flow rate (CalEdPriInt) from the log model of net primary intake rate shown in the earlier figure.

$$CalEdPriInt_{p=male,r,t} = 65.9207 + 7.3423 \ln GDPPCP_{r,t}$$

Subscript *p* in the above equation (and all other equations in this document) stands for sex, *r* stands for countries and *t* for time.

While all countries are expected to follow the regression curve in the long run, the residuals in the base year make it difficult to generate a smooth path with a continuous transition from historical data to regression estimation. We handle this by adjusting regression forecast for country differences using an algorithm that we call "shift factor" algorithm. In the first year of the model run we calculate a shift factor (*EDPriIntNShift*)as the difference (or ratio) between historical data on net primary intake rate (*EDPRIINTN*) and regression prediction for the first year for all countries. As the model runs in subsequent years, these shift factors (or initial ratios) converge to zero or one if it is a ratio (an algorithmic procedure written as a code routine ConvergeOverTime in the equation below) making the country forecast merge with the global function gradually. The period of convergence for the shift factor (*PriIntN_Shift_Time*) is determined through trial and error in each case.

$$EdPriIntNShift_{p,r,t=1} = EDPRIINTN_{p,r,t=1} - CalEdPriInt_{p,r,t=1}$$

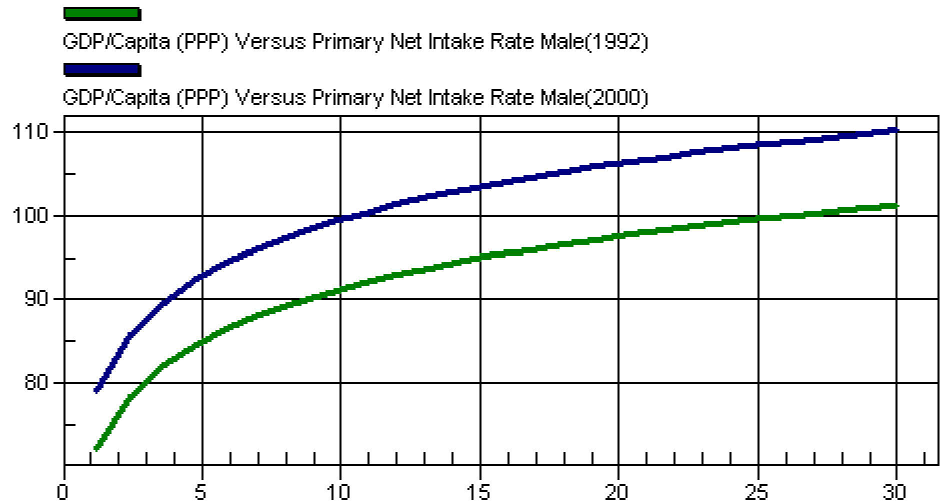
$$EDPRIINTN_{p,r,t} = CalEdPriInt_{p,r,t} + \text{ConvergeOverTime}(EdPriIntNShift_{p,r,t=1}, 0, PriIntN\_Shift\_Time)$$

The base forecast on flow rates resulting from these regression models are added with systemic shift algorithm (see next section) and parameter impacts to calculate the initial or base flow rates. These base flow rates might change as a result of budget impact based on the availability or shortage of education budget explained in the budget flow section.

[3] The name of the equation in the IFs table of functions is “GDP/Capita (PPP 2011) Versus Primary Net Intake Rate Male (MostRecent) Log”

5.3 Education Equations: Student Flow: Systemic Shift

Access and participation in education increase with socio-economic developments that bring changes to people's perception about the value of education. This upward shifts are clearly visible in cross-sectional regression done over two adequately apart points in time. The next figure illustrates such shift by plotting net intake rate for boys at the elementary level against GDP per capita (PPP dollars) for two points in time, 1992 and 2000.



IFs education model introduces an algorithm to represent this shift in the regression functions. This "systemic shift" algorithm starts with two regression functions about 10 to 15 years apart. An additive factor to the flow rate is estimated each year by calculating the flow rate (*CalEdPriInt1* and *CalEdPriInt2* in the equations below) progress required to shift from one function, e.g., $\mathrm{f}_1(GDPPCP_{r,t})$ to the other, $\mathrm{f}_2(GDPPCP_{r,t})$, in a certain number of years (SS_Denom), as shown below. This systemic shift factor (CalEdPriIntFac) is then added to the flow rate (EDPRIINTN in this case) for a particular year (t) calculated from regression and country shift as described in the previous section.

$$CalEdPriInt1_{p,r,t} = \mathrm{f}_1(GDPPCP_{r,t})$$

$$CalEdPriInt2_{p,r,t} = \mathrm{f}_2(GDPPCP_{r,t})$$

$$CalEdPriIntFac_{p,r,t} = \frac{t-1}{SS\_Denom} * (CalEdPriInt2_{p,r,t} - CalEdPriInt1_{p,r,t})$$

$$EDPRIINTN_{p,r,t} = EDPRIINTN_{p,r,t} + CalEdPriIntFac_{p,r,t}$$

As said earlier, Student flow rates are expressed as a percentage of underlying stocks like the number of school age children or number of pupils at a certain grade level. The flow-rate dynamics work in conjunction with population dynamics (modeled inside IFs population module) to forecast enrollment totals.

5.4 Education Equations: Student Flow: Scenario Parameters

Student flow rates calculated from the base model can be changed through parameters. Important among the various parameters described in the scenario manual of the IFs system are multipliers, annual growth parameters and target year parameters. These parameters will show their full impact only when there is no or minimal budget constraint. The budget section of this document and IFs scenario manual explains how one can prioritize education budget over other government expenditure sections.

5.5 Primary Education: Grade Flow Algorithm

Once the core inflow (intake or transition) and outflow (survival or completion) are determined, enrollment is calculated from grade-flows. Our grade-by-grade student flow model therefore uses some simplifying assumptions in its calculations and forecasts. We combine the effects of grade-specific dropout, repetition and reentry into an average cohort-specific grade-to-grade dropout rate, calculated from the survival rate (*EDPRISUR* for primary) of the entering cohort over the entire duration of the level (e.g., ***edprilen*** for primary). Each year the number of new entrants is determined by the forecasts of the intake rate (*EDPRIINT*) and the entrance age population. In successive years, these entrants are moved to the next higher grades, one grade each year, subtracting the grade-to-grade dropout rate (*DropoutRate*). The simulated grade-wise enrollments ($GradeStudents_{d,p,r,t}$ where d is a subscript for the grade level) are then used to determine the total gross enrollment at the particular level of education (*EDPRIENRG* for Primary).

There are some obvious limitations of this simplified approach. While our model effectively includes repeaters, we represent them implicitly (by including them in our grade progression) rather than representing them explicitly as a separate category. Moreover, by setting first grade enrollments to school entrants, we exclude repeating students from the first grade total. On the other hand, the assumption of the same grade-to-grade flow rate across all grades might somewhat over-state enrollment in a typical low-education country, where first grade drop-out rates are typically higher than the drop-out rates in subsequent grades. Since our objective is to forecast enrollment, attainment and associated costs by level rather than by grade, however, we do not lose much information by accounting for the approximate number of school places occupied by the cohorts as they proceed and focusing on accurate representation of total enrollment.

$$DropoutRate_{\mathrm{p,r,t}} = 1 - \left(\frac{EDPRISUR_{p,r,t}}{100}\right)^{\frac{1}{\mathbf{edprilen_r}-1}}$$

$$GradeStudents_{d=1,p,r,t} = EDPRIINT_{p,r,t}$$

$$GradeStudents_{d,p,r,t} = GradeStudents_{d-1,p,r,t-1} * \left(1 - DropoutRate_{p,r,t}\right)$$

$$EDPRIENRG_{p,r,t} = \sum_{d=1}^{\boldsymbol{edprilen_r}} GradeStudents_{d,p,r,t}$$

5.5.1 Primary Education: Gross and Net Flow Rates

Student flow rates. defined as the percentage share of the children of appropriate-age who are in the flow at a particular point in time, can be of two types depending on the age of the student. For example, net enrollment rate in elementary counts only those students who are of elementary-age while the gross elementary enrollment rate includes all pupils in primary regardless of their age in the denominator for the computation of the rate. As the countries with historically low rate of access to education approach a catch-up phase the difference between the gross and the net rates of enrollment, entrance or graduation could be substantial in these countries. Whether and how soon the gross-net gap narrow down in a society depends on the ability and the efforts to expand access. In the current version of the model, we have a full grade-flow model of both the gross enrollment and the net enrollment only for the level of elementary[4].

The model starts with an initial estimation of the pool of out-of-school children for each of the single year age-cohorts in a ten-year age-range starting at the entrance age of primary. These children could either not attend school at all or had to drop out at some point. The estimation is done by subtracting two numbers from the single-year cohort population (*fagedst*):

a. the age-specific enrollment, i.e., those of this single-year cohort who are in school, in an age-appropriate or a higher grade, (*Pristudentsbyage*)
b. age-specific completion, i.e., those, of completion age or older, who have already completed primary

[4] We also have a net enrollment rate forecast for total secondary. That forecast is done through an analytical function driven by the gross enrollment rate in the entire secondary, which is obtained through a properly weighted average of enrollment rates in lower and upper secondary.

The first of these numbers, age-specific enrollment (*PriStudentsbyAge*) is computed by summing up its two parts: those who are regular in entry and progression, and those who has become irregular at some point. The number of regulars is obtained from the grade distribution of the net enrollment (*PristudentsNet*). For the irregulars, we first calculate the number of overage in each grade (*OverAgeInTheGrade*) and then distribute these overage across all single-year cohorts who would be considered over-age for this grade. The distribution uses a normalization algorithm and assumes that the current enrollment rates roughly mimic the age distribution of students. For those who are above the completion age, the enrollment differential (*deltaenr*) between the final and the penultimate grade is used to continue the distribution. As irregulars at all grades are being distributed, the running total of age-specific enrollment rate is updated with the new distribution.

$$ShareNet = EDPRIENRN_{p,r,t}/EDPRIENRG_{p,r,t}$$

$$DeltaEnr = PriStudents_{r,d=\boldsymbol{edprilen_r},p,t=1} - PriStudentsNet_{r,d=\boldsymbol{edprilen_r}-\mathbf{1},p,t=1}$$

$$PriStudents_{r,d=\boldsymbol{edprilen_r}+\mathbf{1},p,t} = f\ (DeltaRnr, ShareNet, EDPRICR_{p,r,t})$$

$$TotOverAgeEnRateTot_d = \sum_{d=d+1}^{\boldsymbol{edprilen_r}} PriStudents_{r,d,p,t} + \sum_{d=\boldsymbol{edprilen_r}+1}^{\mathbf{10}} PriStudents_{r,d,p,t}$$

$$OverageStudentsIntheGrade_{r,d,p,t=1} = PriStudents_{r,d\ \ p,t=1} - PriStudentsNet_{r,d,p,t=1}$$

$$\begin{aligned} PriStudentsByAge_{r,c=\mathbf{1}\ to\ \mathbf{10},p,t=1} &= PriStudentsbyAge_{r,c,p,t=1}^{Calculated\ with\ d-1} + PriStudentsNet_{r,d=\boldsymbol{c},p,t=1} \\ &+ OverageStudentsIntheGrade_{r,d,=c\ p,t=1} \\ &* \frac{PriStudents_{r,d=c,p,t}}{TotOverAgeEnRateTot_d} \end{aligned}$$

Similarly, for completers, part b of the two part listed above, the of-age number of completers is estimated from the gross completion rate (*EDPRICR*) and a ratio of the gross and net enrollment rates (*EDPRIENRN* and *EDPRIENRG*). The rest of the elementary graduates are distributed among those who are older than the completion age but younger enough to return to elementary.

Finally, the in-school (*pristudentsbyage*) and the completers (*prigradbyage*) are subtracted from each of the ten single year cohorts to get the out-of-school children by single-year cohorts (*outofschoolbyage*). Sum of these ten single-year cohorts give an estimate of the pool in the first year of the model.

$$outofschoolbyage_{r,c=\mathbf{1}\ to\ \mathbf{10},p,t=1} = fagedst_{r,\boldsymbol{edpristart_r}+c-1,p,t=1} - PriStudentsByAge_{r,c,p,t=1} - PriGradsByAge_{r,c,p,t=1}$$

Once we have the number of children in the out-of-school pool, we can compute a rate of flow from that pool to the first grade of primary (RetGr1Pcnt) using the initial year difference between the gross and the net entrants as the numerator and the pool headcount as the denominator.

$$OutofSchoolTot = \sum_{1\ to\ 10} outofschoolbyage_{r,c,p,t=1}$$

$$RetGr1Pcnt_{r,,p,t=1} = fagedst_{r,\boldsymbol{edpristart_r},p,t=1} * (PriStudents_{r,d\ \ p,t=1} - PriStudentsNet_{r,d,p,t=1})/outofschoolTot$$

In the subsequent years, the pool is updated from two outflows and two inflows: dropout from schools in the previous year, entrant age children who could not enter school in the previous year, late entry/return to schools in the current year and aging out of children who are no longer young enough to try elementary education.

At first we advance the age of the age-specific out-of-school pool from the previous year. This step takes care of aging out of the eldest cohort from the pool.

$$outofschoolbyage_{r,c=\ 2\ to\ 10,p,t} = outofschoolbyage_{r,c-1,p,t-1}$$

Then, we add those who missed entry as an inflow to the youngest cohort of the out-of-school (*outofschoolbyage* $_{r,1,p,t}$).

$$outofschoolbyage_{r,1,p,t} = fagedst_{r,\boldsymbol{edpristart_r},p,t-1} * (100 - PriStudentsNet_{r,1,p,t-1})/100$$

Next we compute the drop-outs of the previous year and then spread those drop-outs into single-year age cohorts (*Dropoutsfromspread* $_{r,c,p,t-1}$) using a similar normalization algorithm than we have used in the first year to spread all over-age into age-specific cohorts.

$$outofschoolbyage_{r,c=2\ to\ 10\ ,p,t} = outofschoolbyage_{r,c,p,t} + DropoutsFromSpread_{r,c,p,t-1}$$

where

$$DropoutsFromSpread_{r,c,p,t-1} = \mathrm{f}(PriStudents_{r,d,p,t-1}, Dropouts_{r,d,p,t-1}, fagedst_{r,\boldsymbol{c=edpristart_r+1\ to\ 10},p,t-1}$$

The initial rate of return flow (*RetGr1Pcnt*) is converged gradually to 30% in 20 years, numbers we obtained through trial and error, as the model proceeds to the subsequent years.

$$RetGr1Pcnt_{r,,p,t} = ConvergeOverTime(RetGr1Pcnt_{r,,p,t=1}, .3,20)$$

Each year, the elementary entrants who are overage is computed by applying this rate of return to each of the single-year cohorts in the out-of-school pool. The overage-entrant count is then converted to a percentage of the cohort population (*OverageinGr1Pcnt* $_{r,\,p,\,t}$).

$$OverAgeinGr1Count_{r,p,t} = \sum_{1\ to\ 10} outofschoolbyage_{r,c,p,t} * RetGr1Pcnt_{r,,p,t}$$

$$OverAgeinGr1Pcnt_{r,p,t} = 100 * OverAgeinGr1Count_{r,p,t}/fagedst_{r,\boldsymbol{c=edpristart_r},p,t}$$

These over-age entrants are subtracted from each of the single-year cohorts of out-of-school children in the pool.

$$outofschoolbyage_{r,c,p,t} = outofschoolbyage_{r,c,p,t} - outofschoolbyage_{r,c,p,t} * RetGr1Pcnt_{r,,p,t}$$

<u>Grade-flow for Gross Enrollment</u>

The overage entrants computed as a percentage of the entrance age population (*OverageinGr1Pcnt* $_{r,\,p,\,t}$) computed in the pool algorithm is added to the net entrance rate (*EDPRIINTN* $_{r,\,p,\,t}$) to obtain a gross entrance rate.

$$EDPRIINT_{p,r,t} = EDPRIINTN_{p,r,t} + OverAgeinGr1Pcnt_{r,p,t}$$

This gross entrance rate, the survival rate forecast and the number of students in each grade from the previous years are later used to construct the grade-flow for all students. Please see the section on primary education grade flow for further detail on this algorithm.

5.6 Education Equations: Secondary Education

Secondary education is further divided into two levels: a "lower secondary" level with curriculum contents intended to enhance the basic skills obtained in primary and an upper secondary education which is meant to prepare students for college. Both of these levels are three years long, for most countries[5]. Many countries start classifying the students into a general curriculum and a vocational[6] track as soon as they start junior high. IFs education model simulates the lower and upper secondary education of each of the model countries by laying out a system that represents the country specific situation. For example, the cycle lengths for lower (***edseclowrlen***) and upper secondary (***edsecupprlen***) have country specific values initialized with data. Whether a country has vocational education or not and whether the vocational-general split starts at lower secondary or upper, are also modeled according to the nature of the existing system in the country. Since, lower and upper secondary has a very similar algorithm we document below only one of these two levels, i.e., lower secondary and mention the differences between the two levels, when there is any.

5.6.1 Lower Secondary Education: Grade Flow Algorithm

Like elementary, enrollment is the major stock in lower secondary. This stock change through a grade-flow algorithm, again, similar to elementary. Lower secondary students are distributed into the grades of lower secondary as the model starts. In subsequent model time steps, the flows that affect the grade enrollments are:

- an inflow of children who complete primary and transition into the first grade of lower secondary
- dropping out of some of the students from various grades of lower-secondary
- graduation from lower secondary

The table below lists the model variables at the cycle level that represent or determine these stocks and flows.

Variable	Definition	Use
EDSECLOWRENRG	Gross enrollment rate in lower secondary	Stock variable expressed as the rate of participation defined as total students in lower secondary as a percentage share of

[5] 117 of the 186 IFs countries have a three-year lower secondary. Most of the remaining countries have a 4 year lower secondary. Few countries, for example, Germany and Austria, have a unusually long lower secondary cycle of six years. These countries have a shorter elementary cycle, thus keeping the pre-college year total at twelve or thirteen. The number of three-year upper secondary countries is more than 140.

[6] Technical and vocational education track or TVET is the term that UNESCO use

Variable	Definition	Use
		total population in the lower-secondary-age-group
EDSECLOWRTRAN	Rate of transition from primary to lower secondary[7]	This variable determines the inflow to the first grade of lower secondary
EDSECLOWRGRATE	Graduation rate at the lower secondary level	Used in computing the drop-outs and the graduates

Computation of the grade enrollment rates and the total enrollment is shown below. Subscript notation used in these equations have the same meaning as in the other parts of this document (p is for sex, r for country or region, t for time and d for grade, Ages for single-year age cohorts). Intake into the first grade of lower secondary (*caledsecint*) is computed from enrollment rate in the final grade of primary (*pristudents*) and the transition rate into lower secondary (*EDSECLOWRTRAN*) as shown in the first equation. The next equation shows the computation of total cycle drop-outs for this cohort of entrants. The assumptions for this computation is that each of the grades will have the same rate of dropout (*DropoutRate*) and the rate of persistence for the cohort is (roughly) equal to the ratio of the rate of entrance to the rate of graduation (*EDSECLOWRGRATE*). Enrollment rates for the second and higher grades of lower secondary are obtained from the rate of enrollment of the grade below in the year before and the rate of grade drop-out. In a final step, the grade-wise enrollment rates (*seclowrstudents*) are multiplied with population of the relevant cohort ($fagedst_c$*, where c is the subscript for cohort number*) to obtain headcount of students by grade. Grade headcounts are summed to total enrollment in lower secondary (*EDSECLOWRTOT*). The headcount is divided by total number of boys or girls of lower-secondary age-group (*seclowrpop*) and multiplied by one hundred to obtain the enrollment rate.

$$caledsecint_{p,r,t} = pristudents_{edprilen_r,r,t-1} * EDSECLOWRTRAN_{p,r,t}$$

$$DropoutRate_{p,r,t} = 1 - \left(\frac{EDSECLOWRGRATE_{p,r,t}}{caledsecint_{p,r,t}}\right)^{\frac{1}{\boldsymbol{edseclowrlen}_r - 1}}$$

[7] Number of new entrants to the first grade of lower secondary expressed as a percentage of the students enrolled in the last grade of primary in the previous year

$$seclowrstudents_{D=1,p,r,t} = caledsecint_{p,r,t}$$

$$seclowrstudents_{D,p,r,t} = seclowrstudents_{D-1,p,r,t-1} * \left(1 - DropoutRate_{p,r,t}\right)$$

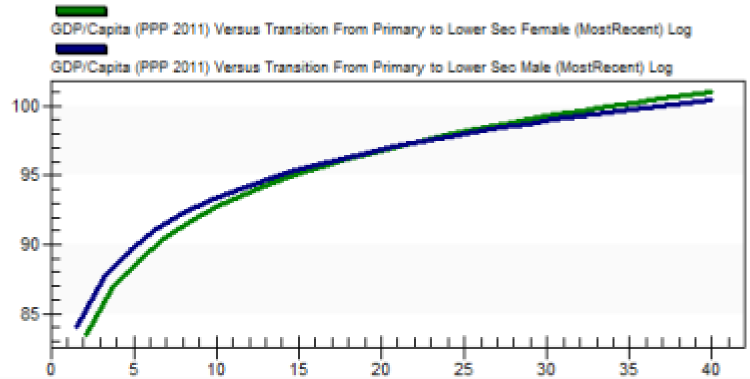
$$edseclowrpop_{p,r,t} = \sum_{c\,=\,\boldsymbol{edpristart_r + edprilen_r}}^{\boldsymbol{edpristart_r + edprilen_r + edseclowrlen_r}} fagedst_{c,p,r,t}$$

$$EDSECLOWRTOT_{p,r,t} = \sum_{d=1}^{\boldsymbol{edseclowrlen_r}} \left(seclowrstudents_{d,p,r,t} * fagedst_{d + edpristart_r + edprilen_r,p,r,t} \right)$$

$$EDSECLOWRENRG_{p,r,t} = 100 * EDSECLOWRTOT_{p,r,t} \,/\, edseclowrpop_{p,r,t}$$

5.6.2 Lower Secondary Education: Key Relationships

Rates of transition into lower secondary (*EDSECLOWRTRAN*) and rates of graduation from lower secondary are driven in the IFs education model by per capita income indicating the level of development of the country and the ability and aspiration of the families. For each of these rates, separate regression equations for boys and girls are estimated from historical data for the most recent year. The regression equations, drawn with most recent historical data, are all logarithmic. The figure below shows the logarithmic functions for the transition rates for the boys and the girls.



Function Name	Function Spec	RSquared
GDP/Capita (PPP 2011) Versus Transition From Primary to Lower Sec Female (Most	Y = 78.9476 + 5.973 LN(X)	0.4033226
GDP/Capita (PPP 2011) Versus Transition From Primary to Lower Sec Male (MostRe	Y = 81.7043 + 5.066 LN(X)	0.3525557

The key variable that drives educational flow rates is the level of development. The flows are first derived from a regression function. The function uses GDP per capita at PPP, computed in the economic model of IFs, as the independent variable. The following is the regression equation[8] used to compute the lower secondary transition rate (CalEdSecLowrTran) for the boys:

$$CalEdSecLowrTran_{p=male,r,t} = 81.7043 + 5.066\ \ln(GDPPCP_{r,t})$$

In the long run all countries converge to the regression curve. The initial country condition is handled by adjusting for country differences computed as a "shift factor". In the first year of the model run the model computes the difference or shift (EDSecLowrTranShift) between historical data (EDSECLOWRTRAN) and regression prediction for the first year for all countries. As the model runs in subsequent years, these shift factors (or initial ratios) converge to zero or one if it is a ratio (code routine ConvergeOverTime in the equation below) making the country forecast merge with the global function gradually. The period of convergence for the shift factor (LowrSecTran_Shift_Time) is determined through trial and error in each case.

$$EdSecLowrTranShift_{p,r,t=1} = EDSECLOWRTRAN_{p,r,t=1} - CalEdSecLowrTran_{p,r,t=1}$$

$$\begin{aligned} &EDSECLOWRTRAN_{g,r,t} \\ &= CalEdSEcLowrTran_{g,r,t} \\ &+ \text{ConvergeOverTime}(EdSecLowrTranShift_{g,r,t=1}, 0, LowrSecTran\_Shift\_Time) \end{aligned}$$

[8] Subscript notations used in this equation are followed throughout this document. Subscript p is used for sex, r stands for countries and t stands for time period in year.

A very similar methodology, with two other regression equations drawn from data, are used for graduation rate in lower secondary. The base forecast on flow rates resulting from these regression models undergo two other adjustment

- Long-run systemic shift (see next section)
- budget impact based on the availability or shortage of education budget explained in the budget flow section.

5.6.3 Lower Secondary Education: Systemic Shift

Educational efforts and outcome increase with socio-economic developments that bring changes to people's perception about the value of education. The next figure illustrates such shift by plotting transition rates in lower secondary for two different points in time.

IFs education model introduces an algorithm to represent this shift in the regression functions. This "systemic shift" algorithm starts with two regression functions about 10 to 15 years apart. An additive factor to the flow rate is estimated each year by calculating the flow rate (CalEdPriInt1 and CalEdPriInt2 in the equations below) progress required to shift from one function, e.g., $\mathrm{f}_1(GDPPCP_{r,t})$ to the other, $\mathrm{f}_2(GDPPCP_{r,t})$, in a certain number of years (SS_Denom), as shown below. This systemic shift factor (*CalEdSecLowrTranFac*) is then added to the flow rate (EDPRIINTN in this case) for a particular year (t) calculated from regression and country shift as described in the previous section.

$$CalEdSecLowrTran1_{p,r,t} = \mathrm{f}_1(GDPPCP_{p,r,t})$$

$$CalEdEdSecLowrTran2_{p,r,t} = \mathrm{f}_2(GDPPCP_{p,r,t})$$

$$CalEdSecLowrTranFac_{p,r,t} = \frac{t-1}{SS\_Denom} * \left(CalEdSecLowrTran2_{p,r,t} - CalEdSecLowrTran1_{p,r,t}\right)$$

$$EDSECLOWRTRAN_{p,r,t} = EDSECLOWRTRAN_{p,r,t} + CalEdSecLowrTran_{p,r,t}$$

As said earlier, Student flow rates are expressed as a percentage of underlying stocks like the number of school age children or number of pupils at a certain grade level. The flow-rate dynamics work in conjunction with population dynamics (modeled inside IFs population module) to forecast enrollment totals.

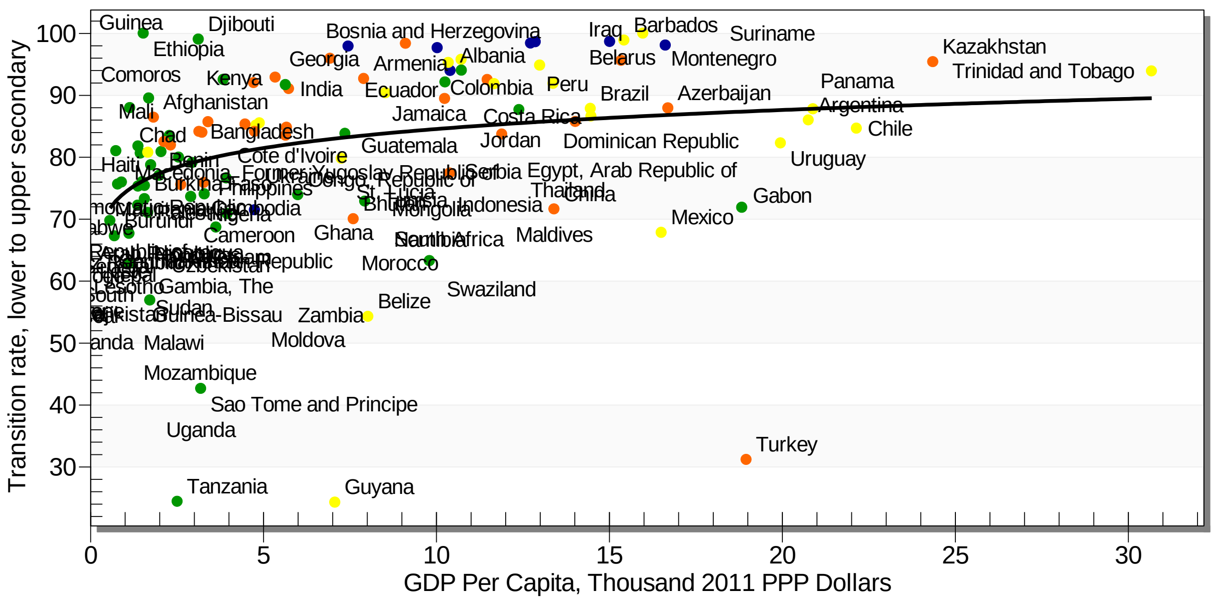
5.7 Upper Secondary Education

Lower secondary completers, whether of-age or over-age, would ideally enroll into upper secondary in the following academic year. In the real world, some of the families might not be able to send their children to school any further. Some who could not continue to upper secondary in the previous years might be able to come back if their difficulties are removed. The metric that we use to compute the gross rate of entrance in upper secondary is the rate of transition from lower secondary to upper[9]. The rate of transition (*EDSECUPPRTRAN*) is defined as the first graders in upper secondary expressed as a percentage share of final graders in lower secondary in the previous year.

Larger shares of completers make the transition as the countries get more developed. The rate of progress slows down at high levels of development as the transition rate moves towards the saturation value of one hundred percent.

Transition from lower to upper secondary vs level of development

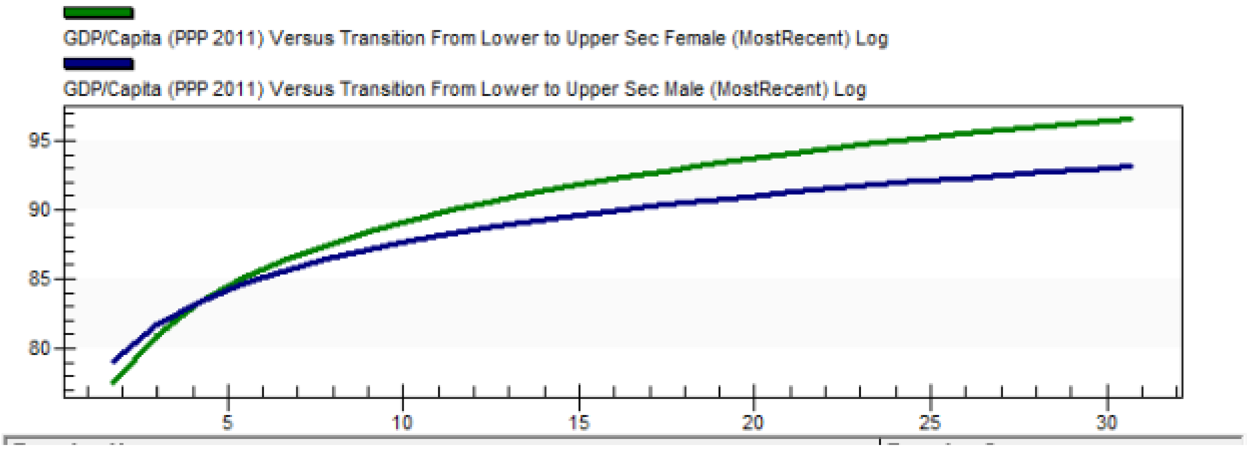
(most recent year)



Two logarithmic functions, one for the boys and one for the girls, obtained from the most recent historical data are used to forecast the transition rate. The shift in gender parity, a phenomenon well observed by now at the upper levels of education in richer countries is quite clear in the

[9] For this series, we use data from UNESCO GMR team's WIDE database

plots where the crossover happens at an income level below $5000 PPP. The regression equations are also given below.



$$CalEdSecUpprTran_{p=male,r,t} = 76.3385 + 4.8996\ \ln(GDPPCP_{r,t})$$

$$CalEdSecUpprTran_{p=female,r,t} = 73.6784 + 6.6943\ \ln(GDPPCP_{r,t})$$

The regression results from the global function are adjusted for country conditions. This is done by computing a country-specific initial shift factor, i.e., the distance between the function output and historical data. In the subsequent years a gradually declining portion of this shift factor, computed through a convergence algorithm, is added to the function output. The number of years ($Tran\_Shift\_Time$) by which all countries converge to the function is determined through trial and error by the model developers. Convergence occurs only for countries that are below the expected rate of transition. For countries that are above the expected value, the convergence is rendered virtually ineffective.

$$EdSUTranShift_{p,r} = EDSECUPPRTRAN_{p,r,t=1} - CalEdSecUpprTran_{p,r,t=1}$$

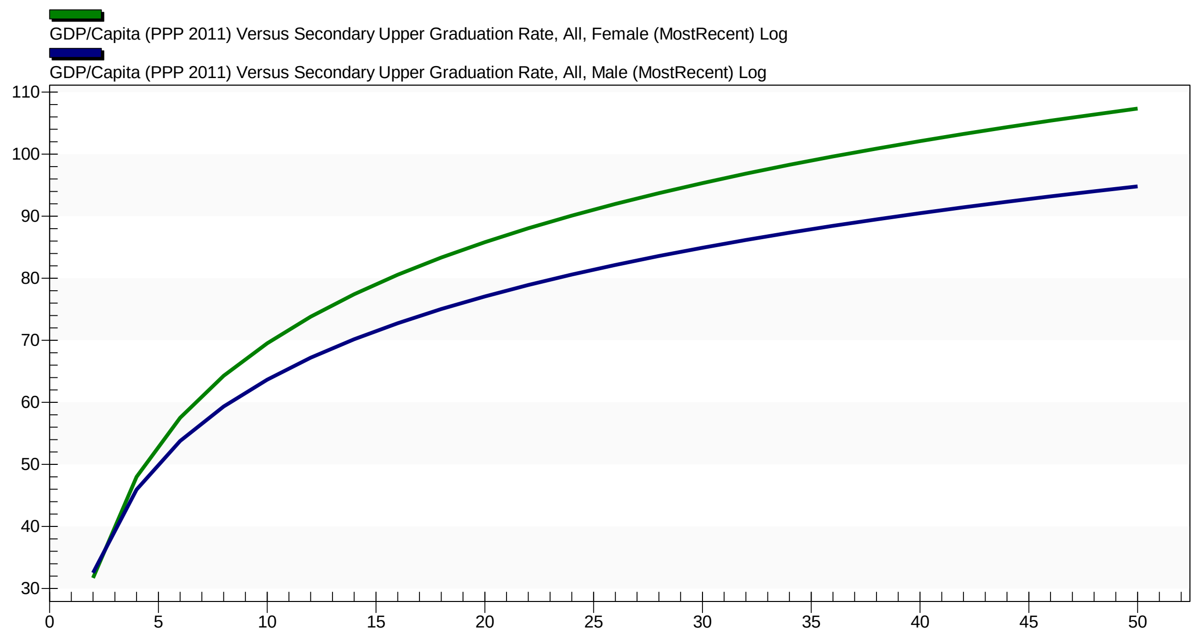
$$\text{If } EdSUTranShift_{p,r} \leq 0, \text{Then}, Tran\_Shift\_Time = 15$$

$$\text{If } EdSUTranShift_{p,r} > 0, Tran\_Shift\_Time = 500$$

$$EDSECUPPRTRAN_{p,r,t} = CalEdSecUpprTran_{p,r,t} + \text{ConvergeOverTimeSmooth}(EdSUTranShift_{p,r}, 0, Tran\_Shift\_Time)$$

The transition rate thus calculated indicates the demand for upper secondary education. Whether the demand is met or not is a question settled through the availability of funds, a process described in the section on education financing.

The other key dynamic relationship in the upper secondary model is the one that drives the rate of graduation. This rate is also a function of per capita income. The two regression functions, one for the boys and the other for girls, are plotted below. Unlike transition, the graduation rate is a gross rate computed as the number of completers, of any age, expressed as a percentage share of the population of the upper secondary completion age. Once again, the girls get ahead of the boys as the country climbs towards an annual per capita income of around $5000.



The regression equations (see below) and the convergence to country specific initial shift as well as the budget impact (described in a latter section) work the same way as in the case of transition rate.

$$CalEdSecUpprGrad_{p=male,r,t} = 19.106 + 19.3498\ \ln(GDPPCP_{r,t})$$

$$CalEdSecUpprGrad_{p=female,r,t} = 15.4152 + 23.4995\ \ln(GDPPCP_{r,t})$$

$$EdSecUpprGRateShift_{p,r} = EDSECUPPRGRATE_{p,r,t=1} - CalEdSecUpprGrad_{p,r,t=1}$$

$$\text{If } EdSecUpprGRateShift_{p,r} \leq 0, Grad\_Shift\_Time = 15$$

$$\text{If } EdSecUpprGRateShift_{p,r} > 0, Grad\_Shift\_Time = 500$$

$$\begin{aligned}\text{EDSECUPPRGRATE}_{g,r,t} \\ &= \text{CalEdSecUpprGrad}_{g,r,t} \\ &+ ConvergeOverTimeSmooth(\text{EdSecUpprGRateShift}_{g,r}, 0, \text{Grad\_Shift\_Time})\end{aligned}$$

Transition rate (*EDSECUPPRTRAN*) is used to deduce the rate of entrance into upper secondary (*caledsecint*) and the gender-specific graduation rate (*EDSECUPPRGRATE*) helps compute drop-out rates by gender. The computation steps are shown below. Subscript notation used in these equations have the same meaning as in the other parts of this document (p is for gender, r for country or region, t for time and d for grade, c for single-year age cohorts).

$$caledsecint_{p,r,t} = seclowrstudents_{edseclowrlen_r,r,t-1} * EDSECUPPRTRAN_{p,r,t}$$

$$DropoutRate_{p,r,t} = 1 - \left(\frac{EDSECUPPRGRATE_{p,r,t}}{caledsecint_{p,r,t}}\right)^{\frac{1}{\boldsymbol{edsecupprlen}_r - 1}}$$

$$secupprstudents_{d=1,p,r,t} = caledsecint_{p,r,t}$$

$$secupprstudents_{d,p,r,t} = secupprstudents_{d-1,p,r,t-1} * \left(1 - DropoutRate_{p,r,t}\right)$$

$$edsecupprpop_{p,r,t} = \sum_{c\,=\,\boldsymbol{edpristart}_r + \boldsymbol{edprilen}_r + \boldsymbol{edseclowrlen}_r}^{\boldsymbol{edpristart}_r + \boldsymbol{edprilen}_r + \boldsymbol{edseclowrlen}_r + \boldsymbol{edsecupprlen}_r} fagedst_{c,p,r,t}$$

$$\begin{aligned}EDSECUPPRTOT_{p,r,t} \\ &= \sum_{d=1}^{\boldsymbol{edsecupprlen}_r} (\, secupprstudents_{d,p,r,t} \\ &* \; fagedst_{\,d + \, edpristart_r + \, edprilen_r + \, edseclowrlen_r, p,r,t})\end{aligned}$$

$$EDSECUPPRENRG_{p,r,t} = 100 * \; EDSECUPPRTOT_{p,r,t} \, / \, edsecupprpop_{p,r,t}$$

5.8 Secondary Education: Vocational Education

Lower and upper secondary education are further divided into a general and a vocational curriculum. Country specific vocational participation data collected from UNESCO Institute for Statistics do not show any common trend in provision or attainment of vocational education across the world. The existence, expansion or phase-out of vocational education in a country results from policy decisions that cannot be modeled analytically. We implement a simple representation of vocational education in the model through a vocational share of total enrollment for lower secondary (*EDSECLOWRVOC*) and upper secondary (*EDSECUPPRVOC*). These vocational shares are initialized with UNESCO data. We assume the vocational shares to be zero when no data is available. The model projects the shares to be constant over time across the entire forecasting horizon.

The vocational shares can be changed through exogenous country-specific parameters, one for lower secondary (*edseclowrvocadd)* and one for upper secondary (*edsecupprvocadd*). These additive parameters can be set to negative or positive values to raise or lower the percentage share of vocational in total enrollment. Changed vocational shares are bound to an upper limit of seventy percent. This upper bound reflects the maximum value of vocational share in the historical data. The upper secondary vocational share in Germany, which at about 67% is the largest among all vocational shares for which we have data.

$$EDSECLOWRVOCI_{p,r} = EDSECLOWRVOC_{p,r,t=1}$$
$$EDSECLOWRVOC_{p,r,t} = \text{Amax}\,(70, EDSECLOWRVOCI_{p,r} + edseclowrvocadd_{p,r,t})$$

5.9 Secondary Education: Total Secondary

The gross enrollment rate in the entire secondary (*EDSECENRG*) is computed by summing up the total enrollment in lower and upper secondary and then dividing that sum by the total secondary-age population (*secpop*).

$$EDSECTOT_{p,r,t} = EDSECLOWRTOT_{p,r,t} + EDSECUPPRTOT_{p,r,t}$$
$$secpop = seclowrpop_{p,r,t} + secupprpop_{p,r,t}$$
$$EDSECENRG_{p,r,t} = (EDSECLOWRTOT_{p,r,t} + EDSECUPPRTOT_{p,r,t})/\text{secpop}$$

Net enrollment rate in secondary is then computed through an analytic function driven by gross enrollment rate.

$$EDSECENRN_{p,r,t} = \text{f}(EDSECENRG_{p,r,t})$$

5.10 Education Equations: Tertiary

The model for tertiary education is slightly different from the grade-flow model used for the elementary and the two secondary levels of education. We could not find any global country-year data series that can help compute the rate of entrance into tertiary. The high tuition and opportunity cost prevent substantial number of low income students from enrolling into college right after high school. Some of those who miss come back for higher education later in life. Moreover, tertiary education has quite some variation across levels and curricula (ISCED has three different levels for tertiary education). All of these phenomena might have made the calculation of an entrance rate difficult for this level. There is, however, good data on participation and graduation. We forecast a gross enrollment rate and a graduation rate (*EDTERGRATE*) for tertiary. A grade-flow distribution is constructed using these two variables[10].

Tertiary enrollment rate (*EDTERENRATEG*) is driven by the level of development and the upper secondary completion rate of those who are twenty to twenty-four years old.

$$CalEdTerEnrG1_{p,r,t} = \mathrm{f}(GDPPCP_{r,t}, EDSECPER20to24_{p,r,t-1})$$

Output from the analytical function is adjusted for country conditions using the same type of shift convergence algorithm that we use for other student flow rates (see, for example, the section on the transition rate from lower to upper secondary).

$$EdTerEnrgShift_{p,r} = EDTERENRG_{p,r,t=1} - CalEdTerEnrg1_{p,r,t=1}$$

$$\text{If } EdTerEnrgShift_{p,r} \leq 0, edterenrgshiftCTime_{p,r} = 25$$

$$\text{If } EdTerEnrgShift_{p,r} > 0, edterenrgshiftCTime_{p,r} = 100$$

$$\begin{aligned} EDTERENRATEG_{p,r,t} & \\ &= CalEdTerEnrg1_{p,r,t} \\ &+ \mathrm{ConvergeOverTime1}(EdTerEnrgShift_{p,r}, 0, edterenrgshiftCTime_{p,r}) \end{aligned}$$

Graduation rate for tertiary is determined by the current rate of enrollment at this level *(EDTERENRATEG)* and the level of tertiary education *(EDTERPER)* of the overall population.

[10] Grade-flow and drop-outs by grade are used in computing education years.

$$CalEdTerGRate_{p,r,t} = \text{f}(EDTERENRATEG_{p,r,t-1}, EDTERPER_{p,r,t-1})$$

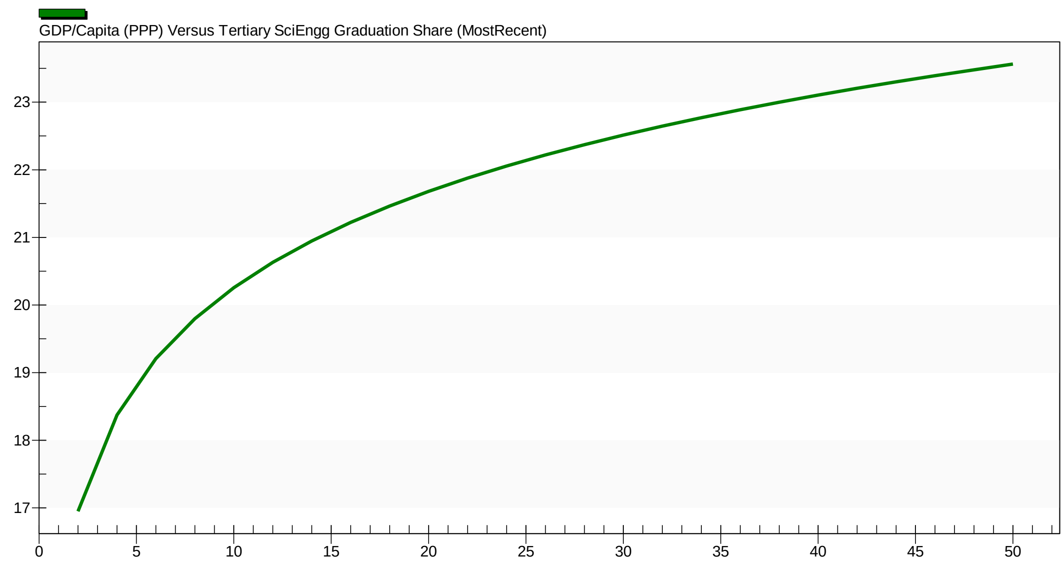
The shift convergence algorithm for the graduation rate is the same as that for the enrollment rate with one difference, the upward convergence time for the graduation rate is 30 years while the downward convergence is virtually blocked.

For approximating the grade-flow algorithm for tertiary we first estimate a dropout rate using the enrollment rate and the graduation rate. Grade-specific enrollment rates are then estimated backward starting at the graduation rate and applying the same dropout rate for all grades.

$$Dropoutrate_{p,r,t} = \text{f}(EDTERENRATEG_{p,r,t}, EDTERGRATE_{p,r,t})$$

5.10.1 Tertiary Education: Science and Technology Graduates

Tertiary study is usually focused on a certain discipline or area of study. Science and engineering is an area considered to be important for the development of a knowledge based society. IFs education model forecasts the percentage share of college graduates[11] who obtain a science and engineering degrees (*EDTERGRSCIEN*). The key relationship is a logarithmic function driven by the level of development.



$$CalEdTerGrSciEn_{r,t} = 15.526 + 2.0543\ \ln(GDPPCP_{r,t})$$

[11] We have not disaggregated this variable by gender yet.

This forecast is also adjusted for country conditions with the initial shift factor and a convergence algorithm like we do with other flow rates.

There is an additive parameter[12] (***edterscienshradd***), with a base case value of zero, that can be used tochange the percentage share of science and engineering among tertiary graduates. This parameter does not have any effect on the total number of tertiary graduates (*EDTERGRADS*).

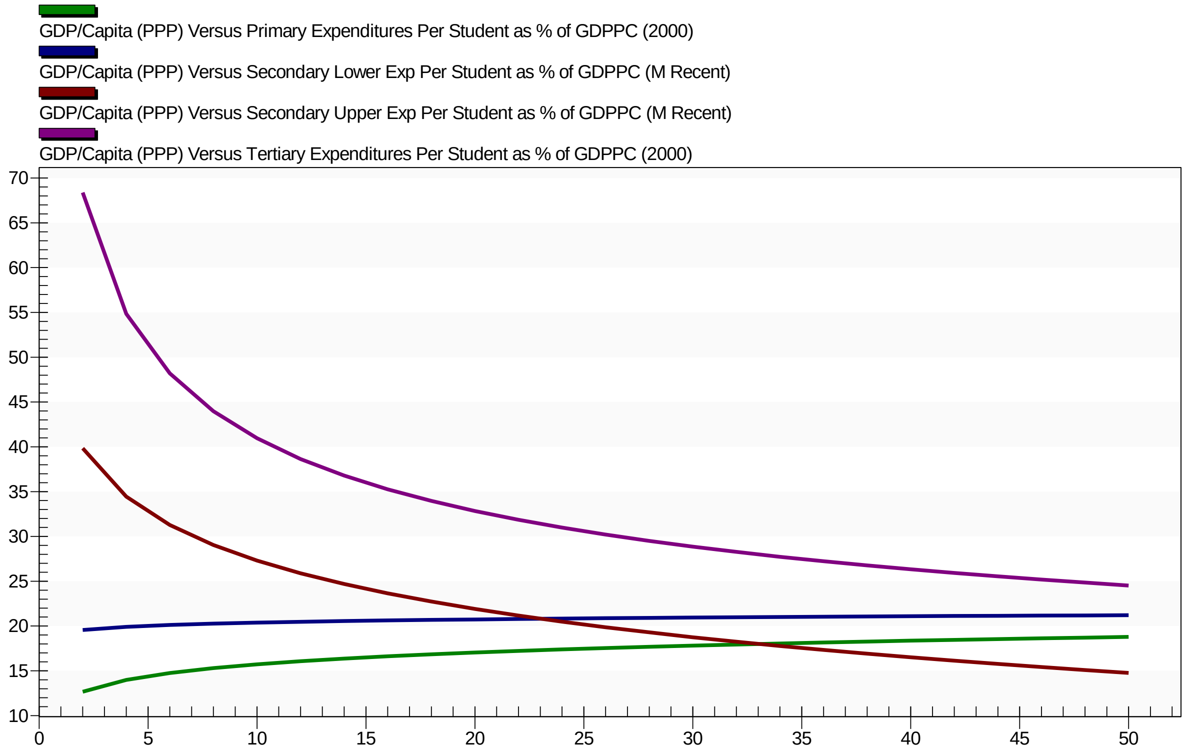
5.11 Education Equations: Budget Flow

Education model computes the demand for funds, using student headcounts and per student costs, and send the demand (*EDBUDDEM*) to the government finance model. Government finance model handles the distribution of government consumption spending across different public sectors like defense, health, education, R&D and infrastructure. Back in the education model, total educational allocation is distributed among the four different levels of education using a normalization algorithm. Deficit or surplus, if there is any, first impacts student flow rates; the grade-flows are adjusted accordingly. Any residual of the budget impact goes to per student costs. Total spending in education (*EDTOTCOST*) is recomputed at the end of this process.

5.11.1 Per Student Cost

The major portion of public spending in education goes towards teacher salary. Salaries move with the average income in a country. Per student public costs for the four different levels of education, expressed as percentages of per capita income, change in our model through four regression functions drawn with most recent historical data. The independent variable for all of these bi-variate regressions is per capita income expressed in thousand PPP dollars. The figure below plots all these functions.

[12] The parameter seems to be inactive (version 7.30)



The following set of equations show the computation of per student costs using the cost in elementary (*EDEXPPERPRI*) as an illustrative example[13].

$$CalExpPerStud_{r,t} = 11.355 + 1.8991\ln(GDPPCP_{r,t})$$

$$EdExpPerPriShift_{r} = EDEXPERPRI_{r,t=1} - \ CalExpPerStud_{r,t=1}$$

$$\boldsymbol{edexppconv} = 50$$

$$EDEXPERPRI_{r,t} = \ CalExpPerStud_{r,t} + \text{ConvergeOverTime1}(EdExpPerPriShift_{r}, 0, \boldsymbol{edexppconv})$$

[13] Per student cost in Primary has a parameter for representing cost differentiation between rural and urban schools. Primary pupil teacher ratio, which is a common block variable that retains the same value over time, can also affect cost. Both of these relationships are inactive now.

5.11.2 Budget Demand

Demand for educational spending is computed from the projections of enrollment and per student cost for the four levels of education.

$$ud\_DemandPri_{r,t} = UD\_EDExpPerPri_{r,t} * GDPPCP_{r,t} * \sum_{p=1}^{2} UD\_EnrollCT_{primary,p,r,t}/100$$

$$ud\_DemandSecLowr_{r,t} = UD\_EDExpPerSecLowr_{r,t} * GDPPCP_{r,t} * \sum_{p=1}^{2} UD\_EnrollCT_{seclowr,p,r,t}/100$$

$$ud\_DemandSecUppr_{r,t} = UD\_EDExpPerSecUppr_{r,t} * GDPPCP_{r,t} * \sum_{p=1}^{2} UD\_EnrollCT_{secuppr,p,r,t}/100$$

$$ud\_DemandSec_{r,t} = ud\_DemandSecLowr_{r,t} + ud\_DemandSecUppr_{r,t}$$

$$ud\_DemandTer_{r,t} = UD\_EDExpPerTer_{r,t} * GDPPCP_{r,t} * \sum_{p=1}^{2} UD\_EnrollCT_{tertiary,p,r,t}/100$$

$$EDBUDDEM_{r,t} = ud\_DemandPri_{r,t} + ud\_DemandSec_{r,t} + ud\_DemandTer_{r,t}$$

5.11.3 Budget Allocation Across Sectors of Spending

Total fund demand (*EDBUDDEM*) is passed to the IFs socio-political model where a detail government budget model distributes total government consumption among various public expenditure sectors using a normalization algorithm described in the government finance model documentation. Distribution to education is prioritized in the base case of the model with a protection of at least 40% of the budget demanded through a model parameter (***gdsbudgetprotec***) that can be changed by model user between 0 to 100%.

5.11.3.1 Educational Budget Allocation: Top-Down versus Bottom-up

Another feature of budget distribution is the reconciliation between allocation on education given the level of development of the country, we call it top-down, and the budget demand projected in the education model, we call it bottom-up. The top-down number is obtained from a regression of educational spending (as a percentage share of total *GDP)* driven by per capita income (*GDPPCP*). This is adjusted for country condition using a multiplicative shift factor computed in the first year from the ratios of historical data on spending in education (*GDS*) and function output (*gkcomp*).

$$gkcomp_{r,g=Educ,t} = \mathrm{f}(GDPPCP_{r,t})$$

$$gkri_{r,g=Educ} = GDS_{r,g=Educ,t=1}/GDP_{r,t=1}/gkcomp_{r,g=Educ,t=1}$$

$$gkshift_{r,t,g=Educ} = ConvergeOverTime\left(gkri_{r,g=Educ}, 75{,}1\right)$$

$$GDS_{r,g=Educ,t} = gkcomp_{r,g=Educ,t} * gkshift_{r,t,g=Educ} * GDP_{r,t}/100$$

There is a lag of one year between the budget demand calculated in the education model and the use of the demand in the government finance model. This lag is a code-sequence issue and compensated through a growth rate term (*EcGrTerm*).

$$sEdTotCost_{r,t} = EDBUDDEM_{r,t-1}$$

$$DemandCalc = sEdTotCost_{r,t} \; * \left(1 + EcGrTerm_{r,t}\right) \; * \; EdCostGDSEdRI_{r,t}$$

where,

$$EcGrTerm_{r,t} = \mathrm{ConvergeoverTime}(igdpr_{r}, 0\;, 50)$$

And EdCostGDSEdRI is a term representing the initial ratio of the bottom-up and the top-down calculation.

5.11.3.2 Educational Budget Prioritization

A normalization algorithm is used to distribute the total available government consumption budget (*GOVCON*) among all sector spending (*GDS, g is the subscript for spending sectors*). Prior to the normalization, a priority parameter (***gdsbudgetprotec***) allows setting aside all or part of demanded funds for the spending sectors. Forty percent of educational demands are set aside in the model base case.

$$GTOT = \sum_{g=1}^{NGovExp} GDS_{r,g,t}$$

$$GDSSetAside_{r,g,t} = GDS_{r,g,t} * (1 - gdsbudgetprotec_{r,g,t})$$

$$TotSetAside = \sum_{g=1}^{NGovExp} GDSSetAside_{r,g,t}$$

$$GTOT = GTOT - TotSetAside$$

$$GovConRed = GOVCON_{r,g,t} - TotSetAside$$

$$GDS_{r,g,t} = GDS_{r,g,t} - GDSSetAside_{r,g,t}$$

$$GDS_{r,g,t} = \frac{GDS_{r,g,t}}{GTOT} * GovConRed + GDSSetAside_{r,g,t}$$

5.11.4 Budget Allocation Across Levels of Spending

Back in the education model, public funding for education ($GDS_{g=Educ}$), undergoes a further normalization across the four levels of education. First, total expenditure demand for all levels of education combined is determined by multiplying the total enrollments with per student costs. The following equation shows the calculation for level of primary.

$$DemandPri_{r,t} = EDEXPERPRI_{r,t} * GDPPCP_{r,t} * (\sum_{p=1}^{2} EDPRITOT_{p.r,t})/100$$

$$DemandSecLowr_{r,t} = EDEXPERSECLOWR_{r,t} * GDPPCP_{r,t} * (\sum_{p=1}^{2} EDSECLOWRTOT_{p,r,t})/100$$

$$DemandSecUppr_{r,t} = EDEXPERSECUppr_{r,t} * GDPPCP_{r,t} * (\sum_{p=1}^{2} EDSECUPPRTOT_{p,r,t})/100$$

$$DemandTer_{r,t} = EDEXPERTER_{r,t} * GDPPCP_{r,t} * (\sum_{p=1}^{2} EDTERTOT_{p.r,t})/100$$

$$CalcTotDemand_{r,t} = DemandPri_{r,t} + DemandSecLowr_{r,t} + DemandSecUppr_{r,t} + DemandTer_{r,t}$$

$$CalcTotSupply_{r,t} = GDS_{r,g=Educ,t} \;/\; SpendCostRI_{r,t}$$

where, *SpendCostRI* is a factor that adjusts any discrepancy between supply and demand side arising out of historical data. In the long run, the ratio converges to 1.1, rather than 1, to account for educational activities not covered in our model, for example, pre-primary education.

Budget surplus or deficit (*Budgetdiff*) is first used to nudge the per student costs towards the expected level, when the direction of deficit between the aggregate and the unit cost are same. The following equations illustrate this algorithm using per student cost in primary (EDEXPERPRI) and the expected level of primary cost (*calEdExpPerStudPri*) as an example case

$$BudgetDiff = CalcTotSupply_{r,t} - CalcTotDemand_{r,t}$$

If $BudgetDiff > 0$ and $calEdExpPerStudPri > EDEXPERPRI_{r,t}$, $EDEXPERPRI_{r,t} = EDEXPERPRI_{r,t} + \left(calEdExpPerStudPri - EDEXPERPRI_{r,t}\right) * 0.02$

If $BudgetDiff < 0$ and $calEdExpPerStudPri < EDEXPERPRI_{r,t}$, $EDEXPERPRI_{r,t} = EDEXPERPRI_{r,t} - \left(EDEXPERPRI_{r,t} - calEdExpPerStudPri\right) * 0.02$

5.11.5 Budget Impact on Enrollment

The previous step does not reconcile the demand and supply of funds entirely. The remaining of the surplus (or deficit) is balanced in two steps. First, a budget impact ratio (*calcbudgetimpact*), a ratio between the supply and the demand of funds, is computed for each level of education. That ratio boosts or reduces the pre-budget access and progression rates. The changed flow rates are then used to compute the total enrollment. As a last step, the final enrollments and the allocated budget are used to revise the per student cost to balance the budget. The equations below show

Budget impacts uses a non-linear algorithm intended to generate an S-shaped growth rate. Final enrollment is then calculated from this final flow rates and any of the remaining budget is used to increase per student expenditure.

$$\mathrm{Cal}cTotCost = (EDEXPERPRI_r/100) * GDPPC_r * \sum_{p=1}^{2} EDPRITOT_{r,p}$$

$$CalcTotSpend = GDS_{r,g=Educ} * GDSED_{r,Pri}/SpendCostRI$$

$$CalcBudgetImpact = CalcTotSpend/CalcTotCost$$

$$EDPRIINTN_{p,r,t} = \mathrm{f}\left(EDPRIINTN_{p,r,t}, CalcBudgetImpact\right)$$

$$EDPRISUR_{p,r,t} = \mathrm{f}\left(EDPRISUR_{p,r,t}, CalcBudgetImpact\right)$$

5.12 Education Equations: Attainment

Educational attainment forecasts fall into two groups. In one group are the variables that track the average years of education of the adults, for example, the average years of education of those who are 25 and older (*EDYRSAG25*). We also forecast the shares of population with a certain level of education, for example, the percentage of fifteen year olds and older who have completed at least the level of primary (*EDPRIPER*). All these variables are gender disaggregated. In fact, the model computes age-sex-education distribution for five-year age cohorts for both types of variables. IFs model software includes a visual display of these age-sex-education plots for all the 186 model countries for each of the forecast year. Model user can also look at these "education pyramids" for pre-built or custom group of countries and compare the pyramids over time and across countries.

5.12.1 Distribution by level of education completed

IFs education model uses Barro and Lee (2016) data for the attainment distribution of entire population fifteen years and older. A spread algorithm[14] initializes the attainment distribution for five-year age-sex-education cohorts. The spread algorithm uses the current rate of completion, e.g., primary completion rate, *EDPRICR* for the percentage of 15+ with completed primary, *EDPRIPER*, as the rate of attainment for the youngest of the cohorts, i.e., 15 to 19 year olds. For each of the older cohorts the rate of attainment is obtained by subtracting a delta (*spreadfactor*) from the attainment of the younger cohort. The delta is computed so that the population weighted sum of attainment of all five-year cohorts ($EDPriPopPer_{r.g,c=4to\ 21,t=1}$ etc) turns out to be the same as the overall attainment ($EDPRIPER_{r.g,\ t=1}$) through some iterative adjustment, if needed. The equations below show the algorithm for elementary attainment. The equations for attainments at secondary and tertiary are very similar.

$$spreadfactor = \mathrm{f}\left(EDPRIPER_{r,p,t=1}, EDPRICR_{r,p,t=1}, Agedst_{c=4\ to\ 21,r,p,t=1})\right)$$

$$EDPriPopPer_{c=4,p,r,t=1} = EDPRICR_{r,p,t=1}$$

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[14] The algorithm was first used by Weishuang Qu of the Millennium Institute in their Threshold 21 model. The more recent versions of the Barro and Lee (2016) datasets have attainment data for five-year cohorts. Wittgenstein Center (WIC) for Demography and Global Human Capital at Austria have a similar age-sex-education dataset.

$$EDPriPopPer_{c=5\ to\ 21,p,r,t=1} = EDPriPopPer_{c-1,p,r,t=1} - spreadfactor$$

where, subscript *c* stand for five-year age cohorts going from 1 to 21. Cohort 4, represents the 15 to 19 years and NC, total number of age cohorts.

The educational level of the people in the five-year cohorts do not change over time except for those cohorts that include young graduates[15]. For all other cohorts, the flows that modify the attainment levels are the attainment of those who join the cohort as they get older and the attainment of those who leave the cohort either through aging or death. As the model runs, each year we add the graduates to the appropriate young cohort. We then modify the rates of attainment of all cohorts including the one that contains the graduates by adding one fifth of the previous year's rate of attainment of the immediate junior cohort and subtracting one fifth of the attainment rate of the cohort in question[16]. A population weighted average of the cohort attainments give the attainment for the entire adult population. Once again we illustrate with examples from attainment of elementary. Cohort attainments for secondary and tertiary education (*EDSECPOPPER*, *EDTERPOPPER*) are initialized and forecast in a similar fashion.

$$EDPriPopPer_{c=4,p,r,t} = \left(\frac{4}{5}\right) * \ EDPriPopPer_{c=4,p,r,t-1} + \left(\frac{1}{5}\right) * EDPRICR_{r,p,t}$$

$$EdPriPopPer_{c=5\ to\ 21,p,r,t} = \left(\frac{4}{5}\right) * EdPriPopPer_{c,p,r,t-1} + (\frac{1}{5}) * EdPriPopPer_{c-1,p,r,t-1}$$

$$EDPRIPER_{r,p,t} = \frac{\sum_{c=4}^{21} EdPriPopPer_{c,p,r,t} * \ Agedst_{c,p,r,t}}{\sum_{c=4}^{NCohorts} Agedst_{c,p,r,t}}$$

5.12.2 Average Years of Education

Average years of education is computed by adding the "level-completion" years with the "partial-years." Computation of the first one obtained from a population weighted average of total years of education for the people who have completed a certain level of education and the length of the level in years. For partial years we can only use the years of education of those who are dropping out from the current system. These estimations do not always match with historical data. We save the discrepancy in the base year and keep on adding a gradually disappearing share of that factor to the computation in the subsequent years. Below is the equation for

[15] We are assuming that the share of those among the graduate who are several years older than the right age of graduation is very small and can be ignored. We also assume that the people retain their formal education throughout life even though we know that it's the credential that is retained not the skills, which can go up or down depending on experience and training, things that we do not model.

[16] The education model uses rates of attainment, which it applies on the population of the cohort, computed in the demographic model. Mortality is taken care of in the population model.

computing average years using elementary education as an example **(edprilen** $_{r,t}$ is the duration of primary cycle in years).

$$AvgYearsPriEdPop_{p,r,t} = \frac{\sum_{c=4}^{NCohorts} \frac{EDPriPopPer_{c,p,r,t}}{100} * \boldsymbol{edprilen_r} * AGEDST_{c,p,r,t}}{\sum_{c=4}^{NCohorts} AgeDst_{c,p,r,t}}$$

For those who dropout before completing a certain level we need to calculate the partial attainment and add that to the average years of education. The average of the partial years of education at a particular year is calculated from dropouts by level and grade as shown below. Calculation of the average of partial years resulting from dropouts in primary education is illustrated in the equations below. Partial years from current year dropouts at other levels of education are calculated in the same manner and all the partial years are averaged to an overall average. This new partial attainment is then added to the partial attainment of five year cohorts which are initialized and advanced in a similar manner as that used for cohort averages on completed attainment.

$$DropoutRate_{p,r,t} = \mathrm{f}\left(EDPRISUR_{p,r,t}, \boldsymbol{edprilen_r}\right)$$

$$Gr\_Students_{d,p,r,t} = f\left(EDPRIINT_{g,r,t}, DropoutRate_{g,r,t}, \boldsymbol{edprilen_r}\right)$$

$$PartialPriPersYearsNew_{p,r,t} = \frac{\left(\sum_{d=2}^{\boldsymbol{edprilen_r}} Gr\_Students_{d,p,r,t} * DropoutRate_{p,r,t} * (d-1)\right) * \sum_{c=\boldsymbol{edpristart_r}}^{\boldsymbol{edprilen_r}} fagedst_{c,p,r,t}}{\sum_{c=\boldsymbol{edpristart_r}}^{\boldsymbol{edprilen_r}} fagedst_{c,p,r,t}}$$

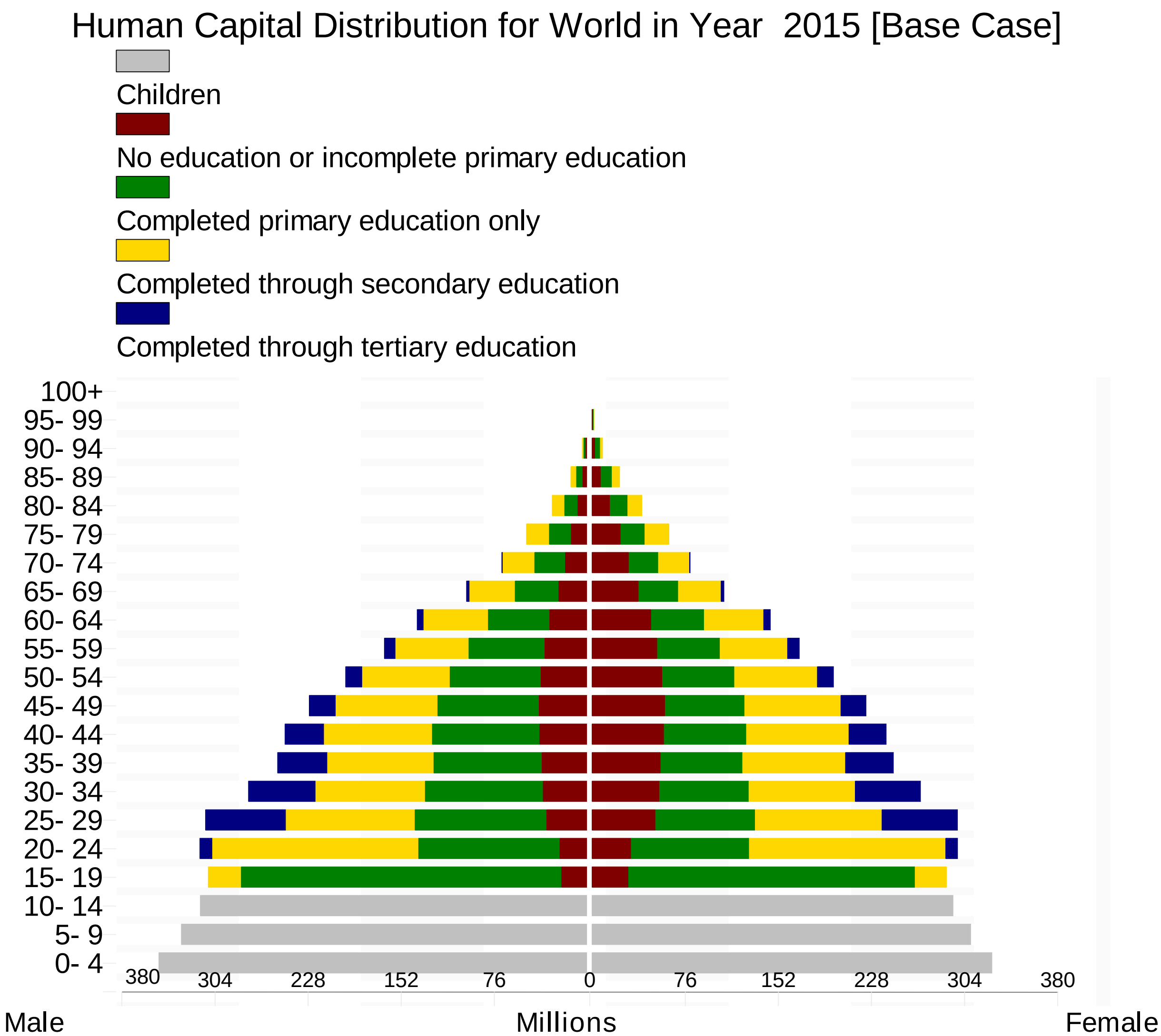
Here, *EDPRISUR* is the survival rate in primary education, ***edpristart*** is the official entrance age for primary schooling, *Gr_Students* is the enrollment at a certain grade, *d* is the grade counter and *fagedst* is the population of the single year age cohort corresponding to the grade level.

Overall attainment, i.e., average years of education are calculated by averaging the attainments and partial attainments of five year age cohorts as shown in the equation below. The suffixes on the variables *EDYRSAG15*, *EDYRSAG15TO24* and *EDYRSAG25* indicate the age thresholds at which or the age bracket over which attainment is averaged.

$$EDYRSAG15_{p,r,t} = AvgYearsPriEdPop_{p,r,t} + AvgYearsSecEdPop_{p,r,t} + AvgYearsTerEdPop_{p,r,t} + PartialYearsEdPop_{p,r,t}$$

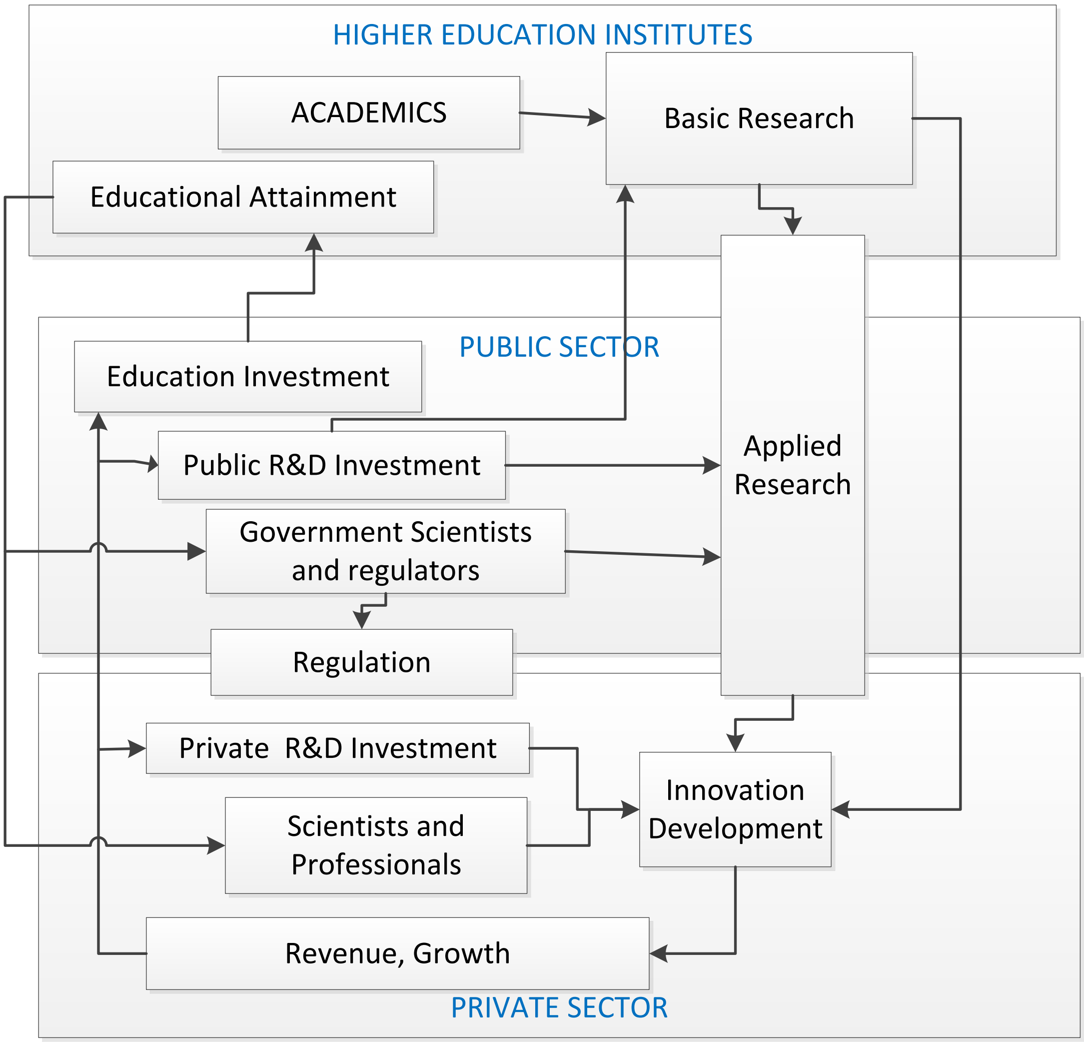
5.12.3 Education Pyramids

Cohort attainments by level of education are used in to build a specialized educational attainment display, commonly referred to as education pyramid in congruence with demographic pyramids used to display population by age cohorts stacked one on top of the other with the men and women cohorts put opposite to each other around a vertical axis. Education pyramid superimposes educational attainment on top of the demographic pyramid.



6 Knowledge Systems

Knowledge and innovation are important drivers of economic growth and human well-being. These activities also help societies address major social and environmental challenges. Education and research and a linear relationship between these and product development are no longer considered a good model of knowledge and innovation systems. However, the linear model was the first successful attempt (Bush, 1945) in conceptualizing the science, technology and innovation (STI) activities. One of the major contributions of these first models was the distinction between basic and applied researches and the identification of stakeholders and funding for each type as shown in the next figure.



The failure of the linear model to capture the intricacies and interactions involved in the innovation process and the broader role of the public and private institutions and individuals in facilitating creation and diffusion of knowledge prompted some experts to resort to rich qualitative description of so called "national systems of innovation" starting from late 1980s, early 1990s. Increased educational attainment, fast expansion of information and communication technologies, more sophisticated production technologies and an expansion in the exchange of goods, ideas and people over the last few decades tell of something broader than just innovation constrained within national boundaries. Recent literature[17] use concepts like knowledge economy or knowledge society to emphasize the recent proliferation of knowledge-intensive activities.

Another important phenomenon discussed in the literature is the systemic nature of the knowledge and innovation and an interlinked emergence of major institutions within such a system (Nelson Ed., 1993). Such a system, known today as the National Innovation System, encompasses major actors, institutions and organizations involved in knowledge and innovation activities and the linkages among them. From a policy perspective, the failures at one or more components of the system, i.e., the systemic failures, justify policy actions more so than the economic rational of market failure. The complementarity of the components of an innovation system demands that the components be studied together. Accordingly, experts have come up with composite indices for assessing the knowledge and innovation capacities of countries around the world. Such indices give a good idea of the overall status of the innovation capacities of the country and the stage of knowledge society it is in. The components of the composite indices are categorized across four to five major dimensions or, pillars, as some studies call these, for example, education and skills, information infrastructure, institutional regime, and innovation activities are some of the pillars used by World Bank's Knowledge Assessment Methodologies (World Bank, 2007).

International Futures (IFs) Knowledge module builds on other knowledge systems measurement approaches by designing a composite knowledge index (KNTOTALINDEX) comprised of five sub-indices containing a total of (x) components. The indices and the sub-indices are then forecast over the entire IFs' horizon by combining the components which are themselves forecast through different modules of the integrated IFs model. To our knowledge, IFs is the only model capable of making such an organic forecast of the knowledge capacity of a country.

6.1 IFs Knowledge Indices:

The capacity of a society to tap from and add to the pool of existing knowledge, local and global, depends on

- skills and qualifications of people to assimilate existing and new knowledge,
- an innovation system to facilitate development or adoption of of new knowledge, processes and products
- a technological infrastructure to share, disseminate and regenerate knowledge and information within and across societies

[17] Peter Drucker popularized the term Knowledge Economy by using it as the title of a chapter in his book the Age of Discontinuity. While the term can cover a wide array of activities the key characteristic of a knowledge economy is a greater reliance on intellectual capabilities compared to that on physical inputs or natural resources (Powell and Snellman, 2014).

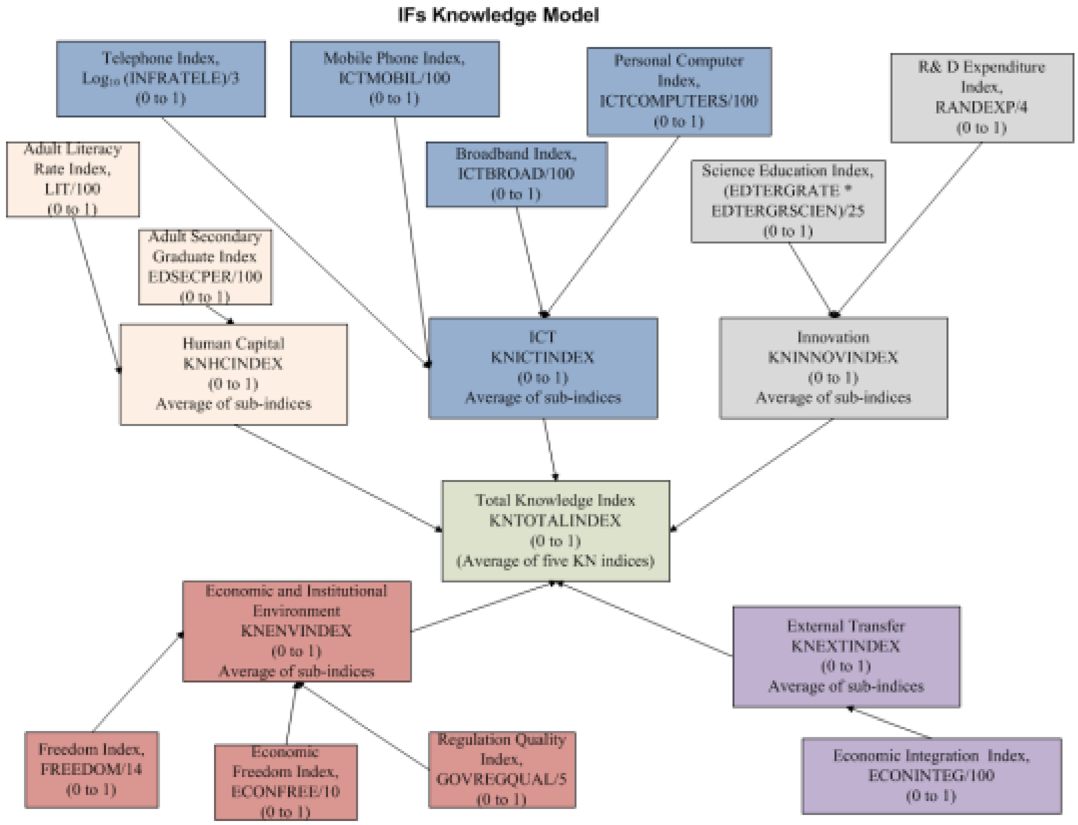
- political and institutional environment conducive to the generation, diffusion and utilization of knowledge
- regulations that offer appropriate incentives towards and remove barriers from international transfer of knowledge

The above list of the driving dimensions of a knowledge system is exhaustive, to the best of our knowledge. The list has five dimensions contrasted to the four pillars identified by the WB KAM. However, World Bank includes tariff & non-tariff barriers, an indicator of international transfer, in their fourth pillar on economic and institutional environment.

IFs now has five indices representing the five dimensions described above. The details of each of these indices, and a sixth one averaged from these five, will be described later. Suffice here to say that, the indices are calculated each of the forecast years by averaging the forecasted value of relevant IFs variables, normalized over a continuous interval going from 0 to 1. That is, IFs integrated simulation, first, forecasts a specific variable, e.g., adult literacy rate, it then converts the forecast to a normalized value lying between zero to one and then averages one or more of these normalized values to obtain an index along each of the dimensions of knowledge assessment. The table below compares IFs knowledge indices with those from World Bank.

No.	Dimension / Pillar	World Bank Variables	IFs Index	IFs variables
1	Human capital	Adult literacy rate; Secondary enrollment rate; Tertiary enrollment rate	KNHCINDEX	Adult literacy rate; Adult secondary graduation rate
2	Innovation	R & D researchers, Patent count; Journal articles (all per million people)	KNINNOVINDEX	Total R & D expenditure (% of GDP); Tertiary graduation rate in science and engineering
3	ICT	Telephones (land + mobile) per 1000 persons; Computers per 1000 persons;	KNICTINDEX	Telephone (fixed); Mobile phone; Personal Computers; Broadband

		Internet users per 10000 persons		
4	Economic and Institutional Regime	Tariff and non-tariff barriers; Regulatory quality; Rule of law	KNENVINDEX	Freedom; Economic freedom; Government regulation quality
5	International Transfer of Knowledge		KNEXTINDEX	Economic integration index
6	Composite Index	Knowledge Index, KI (from the first three) and Knowledge Economy Index, KEI (from all 4)	KNTOTALINDEX	From all of the above



6.2 Knowledge Systems Equations: Total Knowledge Index

The composite index (*KNTOTALINDEX*) consists of five sub-indices, of which the first four contains national actors and institutions only. The fifth one, international transfer index (*KNEXTINDEX*), attempts to capture the impact of global knowledge flows through a measure of the country's openness to the international system. The first four sub-indices - human capital (*KNHCINDEX*), information infrastructure (*KNICTINDEX*), innovation systems (*KNINNOVINDEX*) and governance and business environment (*KNENVINDEX*) – will be described below. The external index (*KNEXTINDEX*) is given a somewhat lower weight in the total index than the other four sub-indices which are equally weighted to a total of 90% of the total index. *KNEXTINDEX* itself is constructed from two equally weighted components of international trade and foreign direct investment.

$$KNTOTALINDEX_{r,t} = 0.9 * \frac{(KNHCINDEX_{r,t} + KNICTINDEX_{r,t} + KNINNOVINDEX_{r,t} + KNENVINDEX_{r,t})}{4} + 0.1 * (KNEXTINDEX_{r,t})$$

6.3 Knowledge Systems Equations: Knowledge Sub-Indices

In this section we describe the calculation method for various IFs knowledge indices.

Human capital Index, KNHCINDEX: The purpose of this index is to capture the cross-country differences in the productive capacity of an average worker. We use two educational stock variables for the purpose. Differences in the rate of literacy, the sheer ability to read or write, make a big difference in productivity in more traditional type and/or informal activities. As the countries move gradually a more traditional agricultural economy to comparatively higher value added activities, e.g., assembling machineries or running a call center, secondary education become more important. The index is built through a combination of two sub-indices: literacy index, LitIndex and secondary attainment index, AdultSecPerIndex, weighted equally.

This index could be improved by adding a measure of the quality of education and an indicator of the skill-base of the worker. Unfortunately, IFs forecasts on those two areas are limited or non-existent at this point. [Note: The sub-indices – LitIndex and AdultSecPerIndex – used for this and other knowledge indices are calculated only in the model code. They are not available for display.]

$$KNHCINDEX_{r,t} = \left(LitIndex_{r,t} + AdultSecPerIndex_{r,t}\right)/2$$

Literacy index, with a theoretical range of values from 0 to 1, is calculated by dividing literacy rate, LIT, which can range from 0 to 100, by 100.

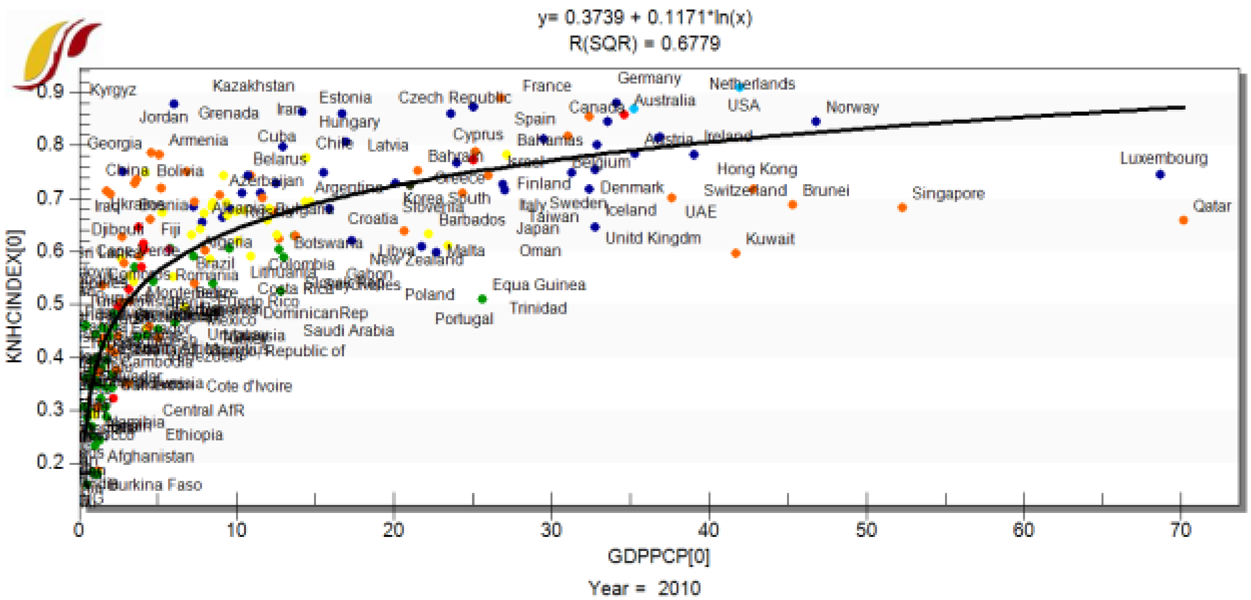
$$LitIndex_{r,t} = LIT_{r,t}/100$$

For the sub-index on secondary attainment (percentage of adults with completed secondary education), we use a similar normalization algorithm like the literacy sub-index.

$$AdultSecPerIndex_{r,t} = EDSECPER_{r,total,t}/100$$

LIT and *EDSECPER* are forecast in the IFs population and education modules.

Because it excludes any measure of higher education which is included in the innovation sub-index (*KNINNOVINDEX*) described below, *KNHCINDEX* turns out to be very useful in showing the differences across developing countries. Even for richer countries, most of which achieved near universal secondary enrollment and universal literacy, the index shows significant variance coming from the secondary attainment differences among the elderly.



Innovation Index, KNINNOVINDEX:

This IFs knowledge sub-index measures the innovation capacity of a nation through its R&D inputs – resources and personnel. It comprises of a total R&D expenditure index and a tertiary science and engineering graduation index as shown in the equations below.

$$KNINNOVINDEX_{r,t} = (RandDExpIndex_{r,t} + EdTerGrateIndex_{r,t})/2$$

For R&D expenditure, the highest spenders like Israel and Finland, spend close to or little over 4% of GDP and we use that number as a maximum to normalize all other countries in a zero to one range.

$$RandDExpIndex_{r,t} = RANDDEXP_{r,t}/4$$

For science and engineering graduation rate, 25% is used as a maximum. The equations below show the calculation which uses tertiary graduation percentage, EDTERGRATE $_{\text{Total}}$ and the share of total graduates that obtain a science or engineering degree, EDTERGRSCIEN, both of which are forecast in the IFs education model.

$$EdTerGrateIndex_{r,t} = EDTERGRATE_{r,total,t} * \frac{EEDTERGRSCIEN_{r,t}}{100}/25$$

ICT Index, KNICTINDEX

Information and communication technologies (ICT) have a very significant role in facilitating the creation and diffusion of knowledge. IFs knowledge sub-index on ICT is built from the diffusion rates of core ICT technologies mobile, landline, broadband and a personal computer access rate sub-index. The telephone lines (fixed lines) sub-index, unlike the other three, use the logarithm of telephone line access rates as the differences in impacts of plain old telephone system decreases at higher access rates. In fact, the gradual shift from a wired to a wireless line as a personal communication device, demands that we reconsider the inclusion of this component in the ICT index.

$$KNICTINDEX_{r,t} = \left(ICTTelephoneIndex_{r,t} + ICTMobileIndex_{r,t} + ICTBroadIndex_{r,t} + ICTComputersIndex_{r,t}\right)/4$$

$$ICTTelephoneIndex_{r,t} = log(INFRATELE_{r,t})\ /3$$

$$ICTMobileIndex_{r,t} = ICTMOBIL_{r,t}/100$$

$$ICTBroadIndex_{r,t} = ICTBROAD_{r,t}/100$$

$$ICTComputersIndex_{r,t} = ICTCOMPUTERS_{r,t}/100$$

Governance and Regulatory Environment, KNENVINDEX:

The existence of economic and regulatory institutions and an effective governance of such institutions are important for generation, diffusion and utilization of knowledge. IFs knowledge sub-index representing these, *KNENVINDEX*, is calculated from three sub-indices which are themselves indices forecast by other IFs modules. These indices, one for economic freedom, a second one for overall freedom in the society and a third one on governance regulatory quality are each normalized to a 0 to 1 scale and averaged to get *KNENVINDEX*.

for the variables economic freedom, political freedom and governance regulation quality and average them to *KNENVINDEX*.

$$KNENVINDEX_{r,t} = \left(EconFreeIndex_{r,t} + FreeDomIndex_{r,t} + GovRegQualIndex_{r,t}\right)/3$$

$$EconFreeIndex_{r,t} = ECONFREE_{r,t}/10$$

$$FreeDomIndex_{r,t} = FREEDOM_{r,t}/14$$

$$GovRegQualIndex_{r,t} = GOVREGQUAL_{r,t}/5$$

International Transfer Index, KNEXTINDEX

KNEXTINDEX attempts to represent cross-national knowledge flows, a major phenomenon in today's globalized world. The more open a country is the more likely it is for her to learn from the global advancements in science, technology and other forms of knowledge. The sub-index that IFs calculates uses two indicators, trade and foreign direct investment (FDI). FDI indicator is given twice the weight given to trade volume.

$$KNEXTINDEX_{r,t} = \left(TradeIndex_{r,t} + 2 * InvIndex_{r,t}\right)/2$$

$$TradeIndex_{r,t} = log\frac{XRPA_{r,t} + MRPA_{r,t}}{GDPPOT_{r,t}} / log\,1000$$

$$InvIndex_{r,t} = log\frac{XFDISTOCK_{r,t} + XFDISTOUT_{r,t}}{GDPPOT_{r,t}} / log\,500$$

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